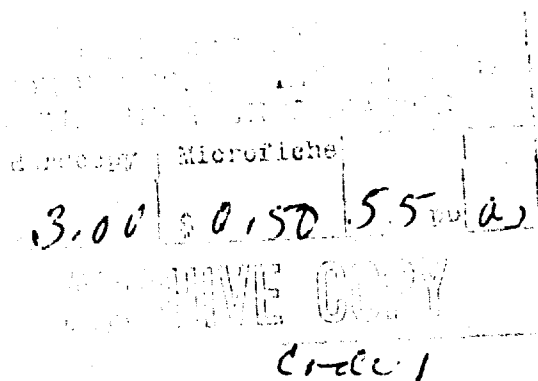# A DESIGN PROCEDURE FOR DETERMINING THE CONTRIBUTION OF DECKHOUSES TO THE LONGITUDINAL STRENGTH OF SHIPS

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A Design Procedure for Determining the Contribution of

Deckhouses to the Longitudinal Strength of Ships*

by

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ABSTRACT

A design procedure developed by the author for determining the stresses in deckhouses is presented. The method includes a tabular procedure for calculating an effective moment of inertia which reflects the effectiveness of the deckhouse in contributing to longitudinal strength. The procedure is based on the theoretical approach by A.J. Johnson of the British Shipbuilding Research Association. The theoretical approach utilizes semi-empirical results of full scale experiments to evaluate the effect of differential deflections between deckhouses and their parent hull girders. This data has been incorporated into an analytical treatment of the problem based on the plane stress theory.

The nature of the problem is discussed and highlights from previously published work on the subject are given. In the conclusions, it is shown that all deckhouses contribute somewhat to the strength of ships and that this fact is useful to the naval architect in his quest for a structurally efficient ship.

INTRODUCTION

Since about 1890, naval architects have calculated what is known as the longitudinal strength of ships by placing the ship on a static "standard wave" which has a length equal to that of the ship and a height from crest to trough of about 1/20 the ship's length. The combination of buoyancy forces and weight forces results in a load diagram from which shear and bending moment curves are calculated. Assuming that the structural behavior of a ship's main hull girder will be similar to a free-free beam, the Naval Architect calculates the stresses by the well known flexure formula, $\sigma = Mz/I$, where σ is the longitudinal stress, M is the bending moment, z is the distance from the neutral axis, and I is the area moment of inertia of the section under consideration. In the inertia calculation, the naval architect has been reluctant to include the deckhouse as a contributing longitudinal strength member.

The reason for this is quite logical. For some time, it has been recognized that the conventional beam theory does not generally apply to the combined deckhouse and hull. In fact, it has been shown that it is possible to have almost any stress distribution in the deckhouse depending on its effectiveness in contributing to the strength of the hull. Vasta [1], in 1949, was the first to demonstrate this with the tests on the S. S. President Wilson.

Realizing that the deckhouse structure may contribute to longitudinal strength, the naval architect, nevertheless, has no way of evaluating its effect. Always conservative in his design, he quite logically omits the deckhouse in his longitudinal strength calculation. In connection with this procedure, there often exists a popular misconception which hypothesizes a dichotomy between "stressed" and "unstressed" deckhouses. If the deckhouse is not considered in the inertia calculation, then it is considered "unstressed." But, in actuality, this would be possible only if the deckhouse were floating on the main deck and completely unattached to the main hull girder. Otherwise, if it is attached in any manner whatsoever, it is experiencing some longitudinal stress, and contributing to the total strength of the ship. This misconception has usually had no ill effects because other considera-

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tions (local loads, etc.) have provided adequate structure in the deckhouse to carry the longitudinal stresses. However, it would be beneficial to obtain a clearer picture of the stresses in the deckhouse so that advantage may be taken of its contribution to longitudinal strength. The design procedure presented in the paper is intended to accomplish this objective.

To determine why the flexure formula cannot be applied to the combined deckhouse and hull, one must examine the assumptions upon which the theory is predicated. The beam theory might be referred to as the approach from the strength of materials point of view. One basic tenet of this approach is that the longitudinal strains in both deckhouse and hull vary linearly and are proportional to the distance from the neutral axis. In the main hull of a ship this is approximately true, as numerous experiments have verified. In addition, the beam theory requires that the deckhouse must be constrained to the hull such that the curvature of the two parts are identical during bending. However, the nature of the interaction between deckhouse and hull is complex and the curvature of the two parts may differ radically when subjected to load. As a result, the strains may not remain linear in the deckhouse and therefore, beam theory will not apply. Full scale tests have verified the nonlinearity of strains in the deckhouse.

That the curvature of the deckhouse may differ from that of the main hull may be seen in Figure 1. The system of shear forces which act at the base of the deckhouse, where it is connected to the hull, are eccentric with respect to the neutral axis of the deckhouse and therefore, there is a tendency for the deckhouse to deflect into a curvature of opposite sign to that of the main hull girder. (Curvature of the main hull girder is measured at the top of the side shell whereas curvature of the deckhouse is measured at the base of the deckhouse.) There is another system of vertical forces which tend to cause the deckhouse to follow the curvature of the hull. Depending on the combination of these systems of forces, the deckhouse may have a curvature differing from that of its main hull girder. The effectiveness of the deckhouse and thus its contribution to the longitudinal strength of the ship will depend to a large extent on how closely or how differently the curvature of the deckhouse resembles that of the hull.

Another phenomenon which occurs in deckhouses which must be accounted for in any complete analysis is known as the "shear lag" effect. It usually occurs in thin plating and concerns

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the uneven distribution of flexural stress in flange members
such as the deckhouse decks. The shear flow, shearing stress,
and shearing strain in the flange plate are higher near the
web (or deckhouse side) than remote from the web. The unequal
shearing deformation causes the section remote from the web to
"lag" as the beam is bent. The result is that plane sections
do not remain plane which denies a basic tenet of beam theory.

Historical Review

Concern over the deckhouse problem dates back to 1899
when Bruhn [2], studying discontinuities in ship structures,
concluded that deckhouse stresses would not approach the simple
beam theory values unless the deckhouse were eight times as
long as high.

In 1913, Foster King [3] presented a paper in which he
used beam theory to determine the stresses in large deckhouses.
His design philosophy was that these stresses should not exceed
those in the main hull girder if the deckhouse were omitted.
Montgomerie [4], in 1915 extended King's treatment and through
an analytical approach attempted to derive rational design
formulas, which were later adopted by some of the Classification
Societies. Expansion joints were introduced as a solution to
the problem of an extremely flexible deckhouse which was unable
to take part in the straining action to which a ship is subjected.

Hovgaard [5], considering a vertical plate of limited
length attached to a horizontal plate of greater length, was
one of the first to recognize the effects of curvature of deck
house and hull. He considered shear in the boundary layer and
concluded from his analysis that expansion joints might aggravate
the stresses rather than relieve them.

The analysis up to and including Hovgaard's work in 1934
appeared to be of a superficial nature. Not until the full scale
experiments of Vasta in 1947, on the S.S. Philip Schuyler [6],
and in 1949, on the S.S. President Wilson [1], did the problem
stimulate comprehensive theoretical attempts. Vasta clarified
the existence of the problem, emphasizing the manner in which
the stresses vary between the main deck and the deckhouse top.
He introduced the concept of deckhouse effectiveness later to
be used by Caldwell. At the same time model experiments by Holt
[7] and Muckle [8] drew additional attention to the problem.
However, there still remained the need of a theory to explain
the observed phenomenon.

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Although Vasta reported definite shear lag effects in his full scale tests, these were not to be considered in perhaps the first comprehensive theoretical treatment of the problem by Crawford [9] in 1950. Crawford examined the equilibrium of vertical forces between deckhouse and hull and the shear forces at the base. He recognized the possibility of differential curvature between the two parts. His analysis concerned single-level deckhouses extending 35 per cent or more of the length of the ship and he assumed the deckhouse was of such dimensions and scantlings that it would behave as a beam. His solution was cumbersome in that it required the solution of ny involved simultaneous equations.

Bleich [10], in 1953, following Crawford's work and attempting to explain the results found on the S.S. President Wilson, was able to express the stresses in a very simple form. He again used the assumption that Navier's hypothesis (beam theory) applied to deckhouse and hull separately. He used the theorem of stationary potential energy (which states that the deformation of any structure is such that the total potential energy of the system is a minimum) to obtain the general Euler differential equations for the deflections of the deckhouse and hull respectively. He wrote these equations using an average deck flexibility constant k . Bleich did not take into account the shear lag effect.

Following Bleich, Terasawa and Yagi [11] used the minimum strain energy principle but developed a method to superpose shear lag effects by using Reissner's [12] least work solution of shear lag problems. The Japanese have studied the deckhouse problem quite comprehensively as one may note from their 60th Anniversary Series [13] published by the Society of Naval Architects of Japan. It would be of interest to see a design procedure based on their studies.

In 1957, three papers appeared simultaneously in England on the deckhouse problem. They were by Chapman [14], Caldwell [15], and Johnson [16]. Chapman's approach was to assume the deckhouse and hull acted separately, each as beams. He considered the deckhouse to be a beam on an elastic foundation and solved the applicable differential equations by relaxation theory. Caldwell and Johnson took a different approach from any of their predecessors and used the plane stress theory. Allowing for shear lag effects and nonlinear strains in the deckhouse, they reasoned that nonlinearity of strains was due to the fact that the plating was very thin in comparison with its overall dimen-

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sions and that the elastic behavior could only be explained by
recourse to the fundamental equations of elasticity. Caldwell's
approach was felt to be more complete than Johnson's in that
he considered rivet slip at the base of the deckhouse where it
was attached to the hull. However, his analysis considered
only single level deckhouses. In addition, Caldwell represented
the external moment by a Fourer Series expansion, a good repre-
sentation, but cumbersome to evaluate in the design office.
Johnson's approach was considered to be the best with respect
to developing a design method. Following his original attempt,
he published two other papers [17], [18] with A. W. Ayling,
which gave additional impetus to the designer wishing to develop
a simplified, quick, design office procedure. Johnson's pro-
cedure was followed by the author in developing a design method.
Details about his method is contained in the following section
of the paper.

Most recently, M.A. Shade [19], of the University of
California published a deckhouse theory which is an extension
of Bleich's theory differing in that shear lag is included,
different structural materials in deckhouse and hull are con-
sidered, and different boundary cond'   ns for the deckhouse
ends are used. Although design curv.   re presented and it
appears that the procedure could be developed for use in the
design office, it is limited to single level deckhouses and
depends upon the evaluation of a deck flexibility factor k.

JOHNSON'S ANALYSIS

To account for any departure from linearity in the long-
itudinal strains in the deckhouse, Johnson [16] used the theory
of elasticity. His method of analysis is based on the plane
stress theory which utilizes the general equations of equili-
brium and compatibility of the theory of elasticity. No
assumption is made regarding the longitudinal strains. Instead
it is required that all forces acting on an elemental particle
of the body be in equilibrium and that the displacements be
compatible with this requirement.

The approach is to use the Airy Stress function to re-
present the stress in a rectangular plate, which is attached
to the hull and is analogous to the deckhouse side. (Figure 2)
The vertical and longitudinal displacements of the plate at the
connection to the hull are made compatible with those produced

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by the flexure of the hull girder. The longitudinal stress
distribution at the midlength of the plate is then obtained.

Consideration is then given to the effect on the stress
caused by attaching a plate to the deckhouse side. (Figure 3.)
The attached plate represents a deck. The effect of many decks
is then considered thus producing an analysis for a multi-level
deckhouse. The stress distributions in the decks themselves
are considered in the light of effective breadths, taking into
account shear lag effects. Finally, empirical data is intro-
duced in order to account for the difference in curvature
between deckhouse and hull. The stress components resulting
from the above considerations are combined in one equation
which gives the stress distribution at the center of the deck-
house.

The basic assumption used throughout Johnson's analysis
is that the shearing stress distribution in the deckhouse at
the connection to any deck varies linearly along the length
of the deckhouse. (Figure 4). In his paper [16], Johnson gives
a comprehensive discussion of the rationale of this assumption.
Experimentally, the assumption may be supported by the tests
on the S.S. Philip Schuyler [5]. In addition to this assumption,
Johnson's analysis is based on idealized deckhouse structure
but these idealizations are accounted for in the development
of the author's design procedure. The idealized deckhouse
structure on which Johnson based his analysis was assumed to
be (1) symmetrically disposed about amidships, (2) possess decks
of equal lengths and widths, and (3) have sides and decks of
constant thickness.

The Governing Equations

Throughout his analysis Johnson makes use of the follow-
ing governing equations of the plane stress theory. A more
detailed analysis is presented in Appendix I. The treatment
given in the appendix is intended for those who wish to know
more about the general approach. Anyone desiring greater
detail is, of course, referred to Johnson's paper [16].

The state of stress in a thin plate can be represented
by Lagrange's equation as :

(1)
$$\frac{\partial^4 \phi}{\partial x^4} + 2\frac{\partial^4 \phi}{\partial x^2 \partial y^2} + \frac{\partial^4 \phi}{\partial y^4} = 0$$

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where ϕ is the Airy Stress function which defines the stresses as follows:

$$\sigma_x = \frac{\partial^2 \phi}{\partial y^2}$$

(2)
$$\sigma_y = \frac{\partial^2 \phi}{\partial x^2}$$

$$\tau_{xy} = -\frac{\partial^2 \phi}{\partial x \partial y}$$

where σ_x is the longitudinal stress, σ_y is the transverse stress, and τ_{xy} is the longitudinal and transverse shearing stress. A solution to Equation [1] may be written as follows:

(3) $\phi = \sum_{1}^{n} \left[A_n \cosh \alpha_n y + B_n \sinh \alpha_n y + C_n y \cosh \alpha_n y + D_n y \sinh \alpha_n y \right] \cos \alpha_n x$

$\alpha_n = \frac{n\pi}{\ell}$ $(n = 1, 2, 3 \ldots)$

A_n, B_n, C_n, D_n, are the arbitrary constants obtained by applying the boundary conditions. The stress function ϕ is, of course, different for the side of the deckhouse and the decks, but the governing equations upon which the analysis is based are the same.

As previously stated, one of the most important considerations in any deckhouse study is the differential deflection between deckhouse and hull. If the curvature of the deckhouse differs radically from that of the hull, the deckhouse will be less effective as a longitudinal strength member. However, as the size of a deckhouse is increased in length and beam the deckhouse more closely represents an extension of the hull. As a result, the deckhouse will be constrained to follow the curvature of the hull more closely and will be more effective as a longitudinal strength member.

Many of the theoretical studies on the deckhouse problem have attempted to solve this problem by including a stiffness modulus of the deck on which the deckhouse rests. However, the definition and evaluation of such a parameter has been a drawback with regard to realistic ship structure. In an attempt to find a practical solution to this problem, Johnson adopted a simplified approach which utilizes empirical data from full scale tests. Assuming the deflected forms of the hull girder and deckhouse are mathematically similar, a deflection coefficient, C, is defined as the ratio of deckhouse deflection to

hull deflection over the length of the deckhouse. To evaluate
C analytically is practically impossible, but one may make some
logical statements about the choice of C and the factors influ-
encing it. The principal factors influencing C are:

(1) Width of deckhouse compared to beam of ship.
(2) Length of deckhouse compared to length of ship.
(3) Lateral stiffness of deck beams and associated plating.
(4) Disposition of bulkheads and stanchions under and adjacent
    to the deckhouse.

Several of these factors are inter-related. For example, there
is some relationship between the number and spacing of bulkheads
to the ratio of length of deckhouse to the length of ship.
Since main transverse bulkheads can be considered points of no
relative deflection between deckhouse and hull, it seems reason-
able that C would approach 1.0 when the length of the deckhouse
approaches the length of the ship. Another important parameter
is the relation between width of deckhouse (b) and beam (B) of
ship. The elastic restraint provided by the transverse frames
and deck plating becomes large when b/B approaches 1.0 and
therefore C also approaches 1.0.

With these considerations, experimental data showed that
the length ratio and beam ratio of deckhouse to hull were the
most influencing factors. Also, since these ratios usually
increase or decrease proportionately C might safely be related
to just one of these ratios. For the purpose of constructing
the design curves, the deflection coefficients were taken from
the full scale experimental results reported in references [17]
and [18] and are:

ℓ/L	0.10	0.20	0.40	0.60	0.80
DEFLECTION COEFFICIENT	- 0.950	- 0.140	0.670	0.970	1.00

Once the value of C is selected, it is applied directly
to the bending component stresses discussed above. The final
equation for the stress distribution in the deckhouse is ex-
pressed in terms of bending and shearing components in the non-
dimensionalized form:

(4) $$\frac{\sigma_x}{g_o} = \frac{\sigma_{x_0}}{g_o} + \frac{\sigma_{x_1}}{g_o}$$

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where σ_x/ζ_0 is the total stress divided by the shearing stress ζ_0 at base and ends of the deckhouse (Figure 4), σ_{x_s}/ζ_0 is the non-dimensionalized shearing stress component, and σ_{x_i}/ζ_0 the non-dimensionalized bending stress component. All are functions of y, the distance from the strength deck to any point in the deckhouse. The final expression for Equation (4), deduced from simple algebraic considerations is:

$$(5) \quad \frac{\sigma_x}{\zeta_0} = \left[\left\{ \frac{\sigma_{x_0}}{\zeta_0} \right\}_{y=0} + \left\{ 1 - 2C \frac{C_D}{d} \right\} \left\{ \frac{\sigma_{x_i}}{\zeta_0} \right\}_{y=0} \right] \frac{\sigma_{x_r}}{\zeta_0} \cdot \frac{1}{\{\sigma_{x_r}/\zeta_0\}_{y=0}}$$
$$+ C \frac{\sigma_{x_i}}{\zeta_0} + \left\{ 2\frac{C_D}{d} - 1 \right\} C \left\{ \frac{\sigma_{x_i}}{\zeta_0} \right\}_{y=0}$$

where C_D is the distance from the strength deck to the neutral axis of the deckhouse, d is the depth of deckhouse, (Figure 9) and σ_{x_r}/ζ_0 is σ_{x_0}/ζ_0 plus the shearing stress effects due to the various decks. The above expression reduces to the following form for any height at the deckhouse midlength:

$$(6) \qquad \frac{\sigma_x}{\zeta_0} = \xi \frac{\sigma_{x_r}}{\zeta_0} + \psi \frac{\sigma_{x_i}}{\zeta_0} + \gamma$$

where ξ, ψ, and υ', are constants and σ_{x_r}/ζ_0 and σ_{x_i}/ζ_0 vary with y, the distance above the strength deck.

THE DESIGN PROCEDURE

Development

Soon after Johnson presented his original analysis [16], he published (with A.W. Ayling) a graphical presentation [18] in which he constructed graphs for seven "basic ships" relating such factors as the ratio of inertia of hull to effective inertia of deckhouse and hull, percentage reduction of stress at strength deck and keel, percentage length of deckhouse and the ratio of stress at the top of the deckhouse to that at the strength deck.

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In the development of the design procedure presented herein, it was felt that this approach could be extended and generalized. The generalization appeared, at first, to be an almost insurmountable task. There were just too many variables to be considered. These included length of deckhouse, length of hull, beam of deckhouse and variation in beam of deckhouse, beam of hull, thicknesses of all plates in the deckhouse, the height between decks and the variation in heights between decks in the deckhouse, the neutral axis of the deckhouse, the neutral axis of the hull, the number of decks in the deckhouse, the inertia of the hull and deckhouse, and finally the deflection coefficients.

A computer study was made of the various parameters involved. The computerization of Johnson's method and the study of the effect of variation of any one parameter, holding the others constant, made it possible to reach conclusions concerning the construction of design curves.

It was found that some variables had greater effects than others, that some could be neglected for the purpose of constructing design charts and that some could be held constant due to the peculiarities of naval ship design. For example, the height between decks was held constant at eight feet since this is applicable to most U.S. Navy ships, except in the way of helicopter hangars and other special arrangements. Two other parameters, the ratio of deck to side thickness and the distance to the neutral axis of the deckhouse, were found to be relatively unimportant with respect to affecting design scantlings. The ratio of deckhouse side thickness to deckhouse deck thickness was taken as 1.0 for the purpose of constructing the design graphs. The other parameter, the distance from the strength deck to the neutral axis of the deckhouse, was taken as 6.6 feet for one level deckhouses and 10.8 feet for two level deckhouses.

Proceeding with the study of parameters, it was found possible to isolate the most important ones and construct the stress ratio curves shown in Figures 5 to 8. The parameters used in these figures are length of ship, length and beam of deckhouse, and distance from the strength deck to the hull neutral axis. Figures 5 and 8 give the ratio of stress at top of the deckhouse to the stress at the strength deck (σ_T/σ_D). This ratio is used in the calculations to determine the effective neutral axis, the effective moment of inertia, and the stresses in deckhouse and hull. These curves were constructed

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for a value of the distance from the strength deck to hull
neutral axis of fifteen feet (C_H 15.0). In order to vary this
parameter, use is made of Figures 6 and 8, modifying the value
of σ_T/σ_D by K_2 or K_4 for the correct value of C_H.

Figures 5 and 7 were based on Johnson's rather involved
analysis based on the plane stress theory. Because his analysis
had certain limitations with regard to symmetry, it was necessary
to obtain design procedures which would not be restricted by
the limitations of the theory. Also, it was found necessary
to make other assumptions to obtain a quick design office pro-
cedure. The assumption regarding the variation in stress from
the strength deck to the top of the deckhouse is an example.

In this regard, it was decided important at the outset
to choose an analysis based on the plane stress theory in order
to accommodate the possibility of nonlinearity of strains in-
cluding shear lag effects. Now that the curves had been con-
structed using this analysis, it was found expeditious at this
point to rationalize a linear distribution of stress between
the possibly nonlinear values of stress at the strength deck
and top of deckhouse. The design philosophy in this case pur-
ports that if the analysis has a realistic approach, and a
solution is obtained based on this approach, then one may make
simplifying assumptions based on this solution which give good
design results. The approximation of linear stress in the
deckhouse based on Johnson's solution for σ_T/σ_D was tested
in several cases and found to give satisfactory results.

Derivation of Formulae

(a) Equivalent Area

After using the curves to obtain a stress ratio
one can readily derive the expressions for Equivalent Area,
Effective neutral axis, and Effective Moment of Inertia from
the elementary principles of mechanics.

The Equivalent Area of the deckhouse, A_{oe}, is defined
as that area which, if multiplied by the stress intensity at
the strength deck, would yield the total longitudinal force in
the deckhouse.

From the diagram in Figure 9 and the definition given
above, the Equivalent Area of the deckhouse may be written as:

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$$(7) \qquad A_{DE} = \int_{A_D} \frac{\sigma}{\sigma_D} \, dA_D$$

where σ is the stress at any height in the deckhouse and σ_D is the stress at the strength deck. It may also be verified from Figure 9 that the stress in the deckhouse may be expressed as:

$$(8) \qquad \sigma = \sigma_D \left[1 + \frac{y}{d} \left(\frac{\sigma_T}{\sigma_D} - 1 \right) \right]$$

where y is measured from the strength deck, positive upward and negative downward and d is the height of the deckhouse. Substituting (8) into (7) and performing the integration over the deckhouse area yields:

$$(9) \qquad A_{DE} = A_D \left[1 + \frac{c_D}{d} \left(\frac{\sigma_T}{\sigma_D} - 1 \right) \right]$$

where A_D is the actual area of the deckhouse adjusted by the shear lag factor in Figure 10 and the Modulus of Elasticity ratio if appropriate, and the stress ratio σ_T / σ_D may be obtained from the curves in Figures 5 and 7. Notice in the above formulas that A_{DE} may take on values greater than or less than A_D depending on whether σ_T / σ_D is greater to or less than one. For this reason we designate A_{DE} as an "equivalent area" rather than "effective area" because effective usually implies a value less than the real value.

(b) Effective Neutral Axis

From the diagram in Figure 9, the stress in the hull, σ', can be written as follows:

$$(10) \qquad \sigma' = \sigma_D \left(1 + \frac{y}{c_E} \right)$$

where c_E is the distance from the strength deck to the effective neutral axis of hull area and equivalent deckhouse area.

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For equilibrium of longitudinal forces in the deckhouse and hull, one may write:

$$(11) \qquad \int_{A_D} \sigma \, dA_D \;+\; \int_{A_H} \sigma' \, dA_H \;=\; 0$$

Where A_H is area of the hull. Substitution of (10) into (11) and dividing by σ_D gives:

$$(12) \qquad \int_{A_D} \frac{\sigma}{\sigma_D} \, dA_D \;+\; \int_{A_H} \left(1 + \frac{y}{c_E}\right) dA_H \;=\; 0$$

Performing the integration as indicated and noticing that:

$$(13) \qquad
\begin{aligned}
\int_{A_D} \frac{\sigma}{\sigma_D} \, dA_D &= A_{DE} \\[4pt]
\int_{A_H} dA_H &= A_H \\[4pt]
\frac{1}{c_E} \int_{A_H} y \, dA_H &= -\frac{c_H}{c_E} A_H \quad ,
\end{aligned}$$

the expression for the distance from the strength deck to the Effective Neutral Axis is obtained:

$$(14) \qquad c_E = \frac{A_H \cdot c_H}{A_H + A_{DE}}$$

(c) Effective Moment of Inertia

For equilibrium, the moments of the stresses in hull and deckhouse about the effective neutral axis must be equal to the external moment.

$$(15) \qquad M = \int_{A_D} \sigma \, (y + c_E) \, dA_D \;+\; \int \sigma' (y + c_E) \, dA_H$$

The effective moment of inertia may be expressed as:

$$(16) \qquad I_E = \frac{M c_E}{\sigma_D}$$

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Substitution for M from (15) into (16), leads to:

$$I_E = c_E \int\limits_{A_D} \frac{\sigma}{\sigma_D}(y+c_E)\, dA_D + c_E \int\limits_{A_H}(y+c_E)(1+\frac{y}{c_E})\, dA_H$$

Performing the integration as indicated and noticing that:

$$(17) \qquad \int y^2 dA_H = I_H + c_H^2$$

we obtain:

$$(18) \qquad I_E = c_E\left[\int\limits_{A_D}\frac{\sigma}{\sigma_D}y\, dA_D + c_E A_{DE}\right] + I_H + A_H(c_H-c_E)^2$$

If we let:

$$(19) \qquad m_{DE} = \int\frac{\sigma}{\sigma_D}y\, dA_D + c_E A_{DE}$$

then (18) becomes:

$$(20) \qquad I_E = I_H + A_H(c_H-c_E)^2 + m_{DE}\, c_E$$

The first two terms of this formula represent the moment of inertia of the main hull about the effective neutral axis. The last term represents the inertia contribution of A_{DE}, the equivalent deckhouse area. m_{DE} is the statical moment of the equivalent area of the deckhouse about the effective neutral axis. Carrying out further the integration indicated in (19), we obtain:

$$(21) \qquad m_{DE} = c_D A_D + \frac{1}{d}\left[I_D + A_D c_D^2\right]\left[\frac{\sigma_I}{\sigma_D}-1\right] + c_E A_{DE}$$

Stress Distribution

The stresses at the strength deck and keel are calcu-

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lated by the flexure formula utilizing the Effective Moment of Inertia and the distance to the Effective Neutral Axis:

$$(23) \qquad \sigma_D = \frac{M c_E}{I_E} \quad ; \quad \sigma_K = \frac{M(D-c_s)}{I_E}$$

where D is the depth of the hull. The stress at the top of the deckhouse is calculated by the following relation:

$$(24) \qquad \sigma_T = \left(\frac{\sigma_T}{\sigma_D}\right) \times \sigma_D$$

where σ_T/σ_D is obtained from Figures 5 or 7. The stresses σ_D and σ_K are the values of stress amidship since Johnson's analysis is predicated upon the deckhouse being symmetrical about amidship.

For the longitudinal distribution of stresses in the deckhouse, it would be conservative for design of the hull structure to assume that the stress varies linearly from the maximum calculated at amidships to zero at the ends of the deckhouse. The reduction in stress at the strength deck, realized by considering the contribution of the deckhouse, would then be assumed to vary linearly from a maximum at amidships to zero at the ends of the deckhouse.

Tabular Method

The Equivalent Area, the Effective Neutral Axis, and the Effective Moment of Inertia may be calculated by a tabular procedure as shown in the example, Appendix II. The procedure is very similar to the ordinary tabular procedure used for calculating moments and thus lends itself readily to design office practice.

The procedure is to use a tabular form with the following headings:

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①	②	③	④	⑤	⑥
Deckhouse Components	Actual Area	Factor(s)	Equivalent Area	Lever	Moment
			$A_{DE} = \sum ④$	$c_E = \dfrac{A_H \cdot C_H}{A_H + A_{DE}}$	$A_{DE} \cdot c_E$
					$m_{DE} = \sum ⑥$

The areas of the deckhouse components are multiplied by factors which are described as follows:

(a) **Stress Factor.** The stress factor depends on location of the component with respect to the stress diagram. For the house top, the factor is the stress ratio, σ_T / σ_D , obtained from Figures 5 or 7. For a two level deckhouse, the factor for the lower level deck is adjusted to suit the straight line variation in stress (see example). For the sides, the factor is the average value of the stress ratio between top and bottom of the deckhouse, $0.5\,(\sigma_T/\sigma_D + 1.0)$.

(b) **Modulus of Elasticity Factor.** For an aluminum alloy deckhouse on a steel hull, the area of the aluminum deckhouse is reduced by the ratio of the modului, $10 \times 10^6 / 29.6 \times 10^6 = 0.34$

(c) **Shear Lag Factor.** Figure 10 gives the reduction factor for various ratios of breadth to length (b/ℓ), applicable to the decks of the deckhouse, including the stiffeners attached to these decks.

Each deckhouse component is multiplied by the appropriate factors to obtain the equivalent area for each component and placed in column ④ . The statical moment of the equivalent area, m_{DE} is then obtained by multiplying the equivalent areas of the deckhouse components by levers which depend upon the centroid of the stress distribution over the component. For example, the side plating lever would be located at the

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centroid of the trapezoidal stress diagram:

$$(22) \qquad L_{EVER \atop (SIDE \; \mathbb{C})} = \frac{d}{3} \left[\frac{1.0 + 2\left(\frac{\sigma_T}{\sigma_D}\right)}{\frac{\sigma_T}{\sigma_D} + 1.0} \right]$$

For the top of the deckhouse, the lever would be the distance
from the strength deck to the deckhouse top.

The sum of column ④ yields the Equivalent Area, A_{De} .
Once A_{De} has been determined, c_E may be obtained from Equation
(14). The value of A_{De} is then multiplied by c_E and included
in column ⑥ . The sum of column ⑥ yields m_{De} . The general
use of the tabular procedure is straightforward and its use
is best demonstrated by the example shown in Appendix II.

Design Simplifications

As stated previously, Johnson's analysis had certain
limitations with respect to symmetry. His analysis was based
on deckhouses symmetrically disposed about amidships. It was
necessary to obtain design procedures which would not be restrict-
ed by the limitations of the theory.

The basic question to be answered was: If a deckhouse was
unsymmetrically disposed about amidships, how should it be treated?
In other words, how critical was the midship position? If one
examines Johnson's analysis, he finds that the midship section
is critical to some extent. The assumption of linear shearing
stress with a value of zero at the center of the deckhouse im-.
plies symmetrical loading and, as Johnson shows in his second
solution, is equivalent to maximum moment occurring at amid-
ships. Since maximum moment nearly always occurs near amid-
ship the assumption is justified. However, if a deckhouse is
mostly on one side of amidships then it is considered that
the analysis is not applicable.

The solution to this problem was the establishment of
an "effective length" of deckhouse to be determined by design
procedures which are intended as suggestions. In each case,
the design procedures are considered conservative for the main
hull structure. The following procedures are suggested to obtain
an effective length, ℓ_F :

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1) If the deckhouse extends at least $0.25L$ both for-
    ward and aft of amidships, use the actual length of
    the deckhouse.

2) If the deckhouse extends no more than $0.15L$ either
    forward or aft of amidships, take the effective
    length (ℓ_e) of the deckhouse to be twice that of
    the shorter part.

3) If the minimum longitudinal extent either forward
    or aft of the midship section is between $0.25L$
    and $0.15L$ take the effective length (ℓ_E) of the
    deckhouse to be one half of its actual length plus
    the shorter part.

As can be seen in the curves, Figures 5 to 8, the design
procedure is intended to include one and two level deckhouses.
Johnson's analysis was based on deckhouse decks of equal lengths,
heights, and widths, so that again it is necessary to propose
design procedures to bypass the limitations of the theory. The
question arises: When should a second level be considered in
the analysis? Or rephrasing, to a more meaningful question :
When can a second level be considered as contributing to longi-
tudinal strength? Again, the method must be conservative with
a respect to the main hull girder. Figure 11 shows a summary
of a study which was made to give guidance for determining a
rational design procedure. This study was typical of many such
studies made to aid in deciding what design procedures should
be used.

From the summary, it may be seen that the consideration
of two levels at any particular value of ℓ/L will provide
lower allowable design stresses in both hull and deckhouse.
Therefore, since the analysis is based on equal length deck-
houses, it is conservative to disregard an upper level thus
providing higher design allowable stresses in hull and deck-
house. Because the exact effect of including a second level
which is shorter than the first level is not known, it is
desirable to be conservative. Thus, the following design pro-
cedure is formulated with this objective in mind.

1) If the effective length of a second level, con-
    sidered separately, is at least 80% of the effect-
    ive length of the first level, then the second
    level may be included in the effective moment of
    inertia calculation in which case the mean of the

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two effective lengths would be considered the new
effective length of deckhouse for use in Figure 7.

It should be recognized that although we may disregard an upper
level in the analysis, this is not to say that the level may
be assumed to be "unstressed." If an upper level is disre-
garded in the analysis, it is suggested that the scantlings be
designed to the stress at the top of the lower level.

There are numerous little details for which a design
procedure must apply. For example, the breadth of the deck-
house may vary over the deckhouse length. In this procedure,
it is suggested that the mean breadth be used in Figures 5 and
7. However, in the calculation of A_{DG} or m_{DE} the actual
breadths at amidship should be used. A similar procedure is
suggested for plate thicknesses. Use typical thicknesses of
sides and decks, not thicknesses in way of local openings or
other special structure. Openings are not considered in this
analysis; however a deckhouse with many closely spaced large
openings may require some modification to the analysis.

Expansion Joints

Although the use of expansion joints has been popular
in U.S. Navy ships, the present belief is that they should be
avoided whenever possible. It is true that expansion joints,
in effect, change the shearing stress distribution in the deck-
house such each span between joints can be regarded as a
separate deckhouse. As a result, the effective length is con-
siderably shortened and, as can be verified from Figures 5 and
7 a short deckhouse is able to contribute less to longitudinal
strength than a long deckhouse. However, expansion joints cause
stress concentrations at the strength deck which may lead to
cracking. Also, considerable maintenance problems for these
joints have been reported.

Nevertheless, it is conceivable that expansion joints
may be warranted in some cases. Consider the example of a
long, continuous deckhouse which contributes substantially to
longitudinal strength. Perhaps the proportions are such that
the stress in this deckhouse approach the beam theory stresses
for the combined deckhouse and hull. In order to provide for
these stresses, the designer finds he must increase his scant-
lings in the deckhouse to such an extent that the added topside

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weight may become critical. It may then be necessary to add
expansion joints to reduce the stresses and thus reduce the
scantlings to avoid the added topside weight.

However, if the expansion joints are placed judiciously
so as to break the deckhouse into an odd number of equal
lengths, then one section will be symmetrical about amidship,
come within the scope of this analysis, and possibly still con-
tribute substantially to the longitudinal strength of the ship.
If this is the case, it is suggested that the other equal
length sections be designed with similar scantlings as the mid-
ship deckhouse section.

Therefore, to determine if expansion joints are needed
in the deckhouse, it is necessary to calculate the deckhouse
stresses without expansion joints. If these stresses exceed
the desired level, expansion joints may be inserted and the
stresses recalculated.

EXAMPLE

In Appendix II, an example of an analysis of a two level
aluminum deckhouse is given. The effective length of the 01-
level is first calculated. Since the ratio of the shorter part
of the first level to the length of ship is $50/350 = 0.143$,
the effective length, ℓ_E is twice that of the shorter part
($\ell_E = 2 \times 50 = 100 \ FEET$). Since the effective length of the 02-
level is 80 per cent of the effective length of the 01-level,
the 02-level is considered in the analysis. The new effective
length of both levels is then $\ell_E = (80 + 100)/2 = 90 \ FT$. The
stress ratio is obtained from Figure 7 for $\ell_E/L = 0.26$,
and is modified by K_3 from Figure 8. The shear lag factor
obtained from Figure 10 is 0.93 . In the factor(s) column
of the tabular procedure are seen the stress distribution factor,
the shear lag factor, and the ratio of moduli of elasticity
factor. The latter factor transforms the aluminum alloy to an
equivalent steel area. Consequently, the calculated deckhouse
stresses must be multiplied by this ratio to transfer the stresses
back to aluminum alloy stresses. The stress diagram is shown
following the calculations and is compared to the stresses with
deckhouse omitted. The reduction in stress at the strength
deck is not too significant but the example shows that a deck
house which has an effective length of only one-quarter the

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ship's length and is constructed of light material still con-
tributes somewhat to the strength of the hull. In another
calculation, a single level steel deckhouse extending forty-
five per cent of the ships length was shown to reduce the
strength deck stress from 7 t.s.i. to 4.5 t.s.i. reflecting a
substantial contribution to the strength of the hull.

CONCLUSIONS

In order that a deckhouse contribute nothing to the
longitudinal strength of a ship, then the equivalent area,
A_{DE}, must be zero. This may be seen clearly from Equation
(14). If $A_{DE} = 0$, in this expression, then C_E will equal
C_H. If $A_{DE} = 0$, equation [9] becomes

$$\left[1 + \frac{C_D}{d} \left(\frac{\sigma_T}{\sigma_D} - 1 \right) \right] = 0$$

or

(25)
$$\frac{\sigma_T}{\sigma_D} = - \frac{d}{C_D} + 1$$

For nominal values of a single level deckhouse, $d = 8.0$ feet
and $C_D = 6.6$ feet, σ_T / σ_D becomes

$$\frac{\sigma_T}{\sigma_D} = \frac{-80}{66} + 1 = -0.21$$

Since this value of σ_T / σ_D is out of range in Figure 5, it
may be concluded for practical purposes that all single level
deckhouses contribute somewhat to longitudinal strength. For
nominal values of a two level deckhouse, $d = 16$ feet and $C_D = 108$
feet, σ_T / σ_D becomes:

$$\frac{\sigma_T}{\sigma_D} = \frac{-16}{108} + 1 = -0.48$$

Since this value of σ_T / σ_D is out of range in Figure 7, it

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may be concluded that all two level deckhouses contribute to longitudinal strength.

RECAPITULATION

The nature of the problem was discussed dismissing the popular misconception of the dichotomy between "stressed" and "unstressed" deckhouses. A brief review of the literature was given. Johnson's theoretical analysis was chosen because of its approach utilizing the plane stress theory, for its simplified use of empirical data regarding differential deflections between deckhouse and hull, and its adaptability to design office practice. A design method was developed for determining the stresses at the midlength of the deckhouse using Johnson's procedure to construct design curves. Design simplifications were made in order to bypass the limitations of the theory. A first appendix gives the general approach of Johnson's analysis while the second appendix gives a detailed design example of a two level aluminum deckhouse. In the conclusions, it is shown through the derived formulae that all deckhouses contribute somewhat to the longitudinal strength of a ship, and in some cases this contribution may be substantial.

RECOMMENDATIONS

The theoretical approach used to develop the design procedure has been tested in full scale trials reported by Johnson [17, 18]. Agreement between theory and experiment was shown to exist in these tests. However, the design procedure introduced by the author has not been verified by experimental results. It is recommended, therefore, that full scale tests be conducted not only to substantiate the design procedure but to obtain additional data on deflection coefficients. Such tests on a variety of U.S. Naval ships may lead to a better selection of deflection coefficients based on a wide range of geometric parameters. In addition, neither the theoretical approach nor the design procedure give adequate consideration to the design of structure near the ends of the deckhouse. The shear forces are largest in this area and the

use of "wobble plates" to avoid the cracking of plates is popular, but greater attention should be given to this problem.

A recent paper by Shade [19] shows that deckhouse analysis using the Navier hypothesis on the deckhouse and hull separately is still receiving attention. There are those, however, who claim that one must resort to the more basic tenets of the theory of elasticity, namely the plane stress theory. If one approach were tested against the other in a large number of cases, it may be found that the results of both approaches are in agreement for a majority of cases. However, if the contrary is true, the Navier hypothesis approach would appear to be more suspect.

The final resolution, of course, lies with the experimentalist who will hopefully find agreement not only in the theoretical approach but in the design simplifications proposed herein.

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APPENDIX I

JOHNSON'S ANALYSIS

It is the intention of this appendix to give the reader a clear picture of Johnson's approach to the deckhouse problem. It is not intended that the mathematical details be worked out step by step. Those who are inclined to know more about these details are referred to Johnson's paper [16]. It is intended that an overview of the theoretical approach be made available to those who wish to know more about the procedure without plodding through the mathematical details. The governing equations given in the paper are reiterated and the analysis is extended using these equations as a base.

Stress Function

Consider first the rectangular plate attached to the main girder as shown in Figure 1. The state of stress in a thin plate can be represented by Lagrange's Equation as follows:

$$(1) \qquad \frac{\partial^4 \phi}{\partial x^4} + 2\frac{\partial^4 \phi}{\partial x^2 \partial y^2} + \frac{\partial^4 \phi}{\partial y^4} = 0$$

where ϕ is the Airy Stress function which defines the stresses as follows:

$$\sigma_x = \frac{\partial^2 \phi}{\partial y^2}$$

$$(2) \qquad \sigma_y = \frac{\partial^2 \phi}{\partial x^2}$$

$$\tau_{xy} = -\frac{\partial^2 \phi}{\partial x \partial y}$$

where σ_x is the longitudinal stress, σ_y is the transverse stress, and τ_{xy} is the longitudinal and transverse shearing stress. A solution to equation (1) is:

$$(3) \quad \phi = \sum_1^n \left[A_n \cosh \alpha_n y + B_n \sinh \alpha_n y + C_n y \cosh \alpha_n y + D_n y \sinh \alpha_n y \right] \cos \alpha_n x$$

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where

$$\alpha_n = \frac{n\pi}{\ell} \qquad (n = 1, 2, 3 \quad)$$

A_n, B_n, C_n, D_n are the arbitrary constants obtained by applying the boundary conditions. Before stating the boundary conditions, however, it is necessary to write the expressions for longitudinal and transverse displacements of the base of the deckhouse.

Displacements

The displacements are obtained by integrating the expressions for strain which are obtained from the well known relationships given by the theory of elasticity.

(4)
$$\epsilon_x = \frac{\partial u}{\partial x} = \frac{1}{E}(\sigma_x - \nu\sigma_y) = \frac{1}{E}\left(\frac{\partial^2 \phi}{\partial y^2} - \nu\frac{\partial^2 \phi}{\partial x^2}\right)$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \frac{1}{\mu}\tau_{xy} = -\frac{1}{\mu}\frac{\partial^2 \phi}{\partial x \partial y}$$

where ϵ_x is the longitudinal strain, γ_{xy} is the shearing strain, ν is Poisson's ratio, and

$$\mu = \frac{E}{2(1+\nu)}$$

is the shear modulus of rigidity.

Integration of ϵ_x leads to the following expression for u :

(5)
$$u = \frac{4}{3E} \sum_1^n \left(A_n + \frac{3}{2} D_n\right) \sin \alpha_n x \qquad (\nu = 1/3)$$

In this part of the analysis, the deflections at the base of the deckhouse are assumed equal to those of the main hull girder. Later the effect of differential deflections of deckhouse and hull will be brought into the analysis. From the application of the simple beam theory to the hull (Figure 12), we have at the strength deck (where deckhouse meets hull):

$$\frac{\partial^2 v}{\partial x^2} = \frac{1}{R}$$

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where R is the radius of curvature. From (4) we have

$$\epsilon_x = \frac{\partial u}{\partial x}$$

and from Figure 12

$$\ominus \cdot \frac{1}{R} = \frac{\epsilon_x}{-C_H}$$

so that

(6)
$$\frac{\partial u}{\partial x} = \frac{-C_H}{R} = -C_H \frac{\partial^2 v}{\partial x^2}$$

Substitution of [5] into [6] and performing the indicated integration leads to:

(7)
$$V = \frac{4}{3 E C_H} \sum_1^n \left(A_n + \frac{3}{2} \frac{D_n}{\alpha_n} \right) \cos \alpha_n x$$

Before the boundary conditions can be stated, it is necessary to write the expression for shearing stress, which as stated previously is assumed to be linear. As can be seen in Figure 4, the shearing stress distribution is an odd function (i.e. $f(x) = -f(-x)$) and thus may be represented mathematically by a half-range Fourier expansion:

(8)
$$\tau_{xy} = q_0 \sum_1^n \beta \sin \alpha_n x$$

where

$$\beta = \frac{8}{\pi^2} \left(\pm \frac{1}{n^2} \right)$$

In this expression, n is odd and the terms in the expansion are alternately positive and negative.

Boundary Conditions

The boundary conditions needed to obtain the arbitrary constants A_n, B_n, C_n, and D_n are as follows:

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(a) $y = 0$: $\tau_{xy} = q_0 \sum_1^n \beta \sin \alpha_n x$

(b) $y = d$: $\tau_{xy} = 0$

(c) $y = d$: $\sigma_y = 0$

(d) $x = \pm \ell/2$: $\sigma_x = 0$

Since (4) may be satisfied simply by making n odd, an additional boundary condition is needed. This is obtained from the expressions relating strains. By differentiating ϵ_x in (4) with respect to y, and γ_{xy} in (4) with respect to x, we are able to combine the two expressions to obtain an additional boundary condition, which is:

(e)
$$-\frac{1}{\mu}\left(\frac{\partial^2 \phi}{\partial x \partial y}\right) = \frac{1}{E}\left(\frac{\partial^3 \phi}{\partial y^3} - \nu \frac{\partial^3 \phi}{\partial x^2 \partial y}\right) + \frac{\partial^2 v}{\partial x^2}$$

since (7) gives the needed expression for v, it is possible to apply the four boundary conditions and solve for the arbitrary constants A_n, B_n, C_n, and D_n.

Expression for Stress

Solving for the longitudinal stress at the midlength of the deckhouse (x=0), the following expression is obtained.

(10) $\sigma_x = \sum_1^n \alpha_n \left[\left\{ A_n \alpha_n + C_n \alpha_n y + 2 D_n \right\} \cosh \alpha_n y + \left\{ B_n \alpha_n + 2C_n + D_n \alpha_n y \right\} \sinh \alpha_n y \right]$

The above expression conveniently splits into two parts, one for shearing stress components, σ_{x_0}, and the other for bending stress components, σ_{x_1}, which may be expressed non-dimensionally as follows:

(11)
$$\frac{\sigma_{x_0}}{q_0} = \sum_1^n \beta \left[\left\{ A_{(n)_0} + \frac{2}{3}\alpha_n y + 2 D_{(n)_0} \right\} \cosh \alpha_n y + \left\{ \frac{5}{3} + D_{(n)_0} \alpha_n y \right\} \sinh \alpha_n y \right]$$

$$\frac{\sigma_{x_1}}{q_0} = \sum_1^n \alpha_n E v_0 \gamma \left[\left\{ A_{(n)_1} + \frac{1}{2}\alpha_n y + 2 D_{(n)_1} \right\} \cosh \alpha_n y + \left\{ \frac{1}{2} + D_{(n)_1} \alpha_n y \right\} \sinh \alpha_n y \right]$$

where the subscripts indicate a simplified expression for the coefficient, e.g. $A_{(n)_0}$ is a function of A_n involving the

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hyperbolic sine. In his analysis, Johnson expresses the above expression for σ_{x_0}/q_o and τ_{x_1}/q_o in the form of curves for a range of values of c_H/ℓ and y/d. The resultant longitudinal stress at the midlength of the deckhouse for any combination of c_H/ℓ and y/d may be expressed as:

$$(12) \qquad \frac{\sigma_x}{q_o} = \frac{\sigma_{x_1}}{q_o} + \frac{\sigma_{x_2}}{q_o}$$

Restrictive Effect of Deck

Having determined the stress distribution at the midlength of a rectangular plate attached to the hull and analogous to the deckhouse side, Johnson considers the restrictive effect of a deck attached to the side as shown in Figure 2. Assuming linear shearing stress at the connection to the vertical plate, an assumption used previously, Johnson is able to write his solution for stress in the side plate by the use of two different stress functions, one for the side plate above the attachment of the deck and one for the side plate below the attachment.

Boundary Conditions

The arbitrary constants A_n, B_n, C_n, and D_n previously used correspond to G_n, H_n, J_n, and K_n for the upper section and L_n, M_n, N_n, and O_n for the lower section. The arbitrary constants are found by applying the following boundary conditions:

(a) $\quad y = 0 \qquad : \quad \tau_{xy} = 0$

(b) $\quad y = 0 \qquad : \quad \dfrac{\partial^2 Y}{\partial x^2} = 0$

(c) $\quad y = md \qquad : \quad$ LONGITUDINAL DISPLACEMENT $\quad u_U = u_L$

(d) $\quad y = md \qquad : \quad$ VERTICAL DISPLACEMENT $\quad v_U = v_L$

(e) $\quad y = md \qquad . \quad \sigma_{y_U} = \sigma_{y_L}$

(f) $\quad y = d \qquad \quad \sigma_y = 0$

(g) $\quad y = d \qquad : \quad \tau_{xy} = 0$

(h) $\quad x = \pm \ell/2 \quad : \quad \sigma_x = 0$

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where the subscripts U and L refer to the upper and lower section respectively and m is the ratio of deck height to deckhouse height. The condition of a longitudinal stress of zero at the free ends is satisfied by making n odd. Therefore a final boundary condition must be obtained from the equilibrium of longitudinal forces as follows:

$$(12) \quad t_s \left[\int_0^{md} \sigma_{x_L} \, dy + \int_{md}^d \sigma_{x_v} \, dy \right] + t_N q_N \int_x^\ell \sum_1^n \beta \sin \alpha_n x = 0$$

where t_s is the thickness of the side plate and t_N is the thickness of the deck.

Expressions for Stress

Applying the boundary conditions, the following expression for longitudinal stress at the midlength of the deckhouse in the upper section is obtained:

$$(13) \quad \frac{\sigma_x}{r_N q_N} = \sum_1^n \frac{2}{3} \beta \left[\left\{ G_n + y \alpha_n J_n + 2 K_n \right\} \cosh \alpha_n y + \left\{ H_n + y \alpha_n K_n + 2 J_n \right\} \sinh \alpha_n y \right]$$

where r_N is the ratio t_N / t_s. And, for the lower section:

$$(14) \quad \frac{\sigma_x}{r_N q_N} = \sum_1^n \frac{2}{3} \beta \left\{ L_n \cosh \alpha_n y + \left(y \alpha_n \sinh \alpha_n y + 2 \cosh \alpha_n y \right) O_n \right\}$$

Johnson reduces the above expressions to stress functions which are plotted as curves depending upon y/d, b/ℓ, and m.

Effective Breadth

Taking account of the shear lag effect in the decks, Johnson expresses the width of the deck as an equivalent width in which the distribution of longitudinal stress can be taken as uniform and equal to that at the deckhouse side. The equivalent width is denoted as $K_3 b$ where K_3 is the shear lag factor. The uniform effective stress (σ_{x_e}) in the deck is equated to the summation of the actual stress over the deck to obtain an expression for K_3.

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$$K_3 \, b \, \sigma_{x_e} = \int_0^b \sigma_x \, dz$$

The final result is:

(15)
$$K_3 = \frac{\sum_1^n \frac{2\beta}{b\alpha_n}}{4 \sum_1^n \frac{\beta \cosh^2\left(\frac{b\alpha_n}{2}\right)}{\sinh b\alpha_n + b\alpha_n}}$$

The above expression has been evaluated for various values of b/ℓ and plotted in Figure 10.

Superposition of Stresses

Next, by the principle of superposition, Johnson considers the combined restrictive effect of several decks. By equilibrium of longitudinal forces at a deck Johnson obtains the following expression for the resultant stress at any deck:

(16)
$$\sigma_{x_N} = - \frac{q_N \ell}{K_3 \, b_N}$$

From Equations (13) and (14) and their corresponding curves, the general expression for the longitudinal stress at mid-length due to q_N can be written:

(17)
$$\frac{\sigma_x}{r_N \, q_N} = - \phi_N$$

Then applying the principle of superposition, letting σ_{x_0} be the longitudinal stress due to q_0 when no decks are included and σ_{x_A} be the resultant longitudinal stress in the deckhouse at deck A when all decks are included, we obtain:

(18)
$$\sigma_{x_A} = \sigma_{x_{0A}} - \sigma_{x_{AA}} - \sigma_{x_{BA}} - \sigma_{x_{CA}} - \cdots$$

The above expression states that the resultant longitudinal stress at A is equal to the stress at A due to q_0 alone minus the stress at A due to deck A,B,C, etc. From equations

(16) and (17) equation (18) may be expressed as follows:

$$(19) \qquad \frac{q_A \, \ell}{k_3 b_A} = q_0 \, \phi_{oA} - q_A \, r_A \, \phi_{AA} - q_B \, r_B \, \phi_{BA} - q_C \, r_C \, \phi_{CA}$$

or by rearrangement:

$$(20) \qquad \frac{q_A}{q_0}\left[r_A \, \phi_{AA} + \frac{\ell}{k_3 b_A} \right] + \frac{q_B}{q_0} \, r_B \, \phi_{BA} + \frac{q_C}{q_0} \, r_C \, \phi_{CA} + \cdots = \phi_{oA}$$

Similar expressions may be derived for all decks of the deck-house resulting in a set of simultaneous equations in which q_A/q_0, q_B/q_0 etc., are the unknowns. The ϕ terms are obtained from Johnson's curves as explained previously.

Final Expression for Stress

Having obtained through superposition the combined effect of several decks on the stress in the deckhouse side, Johnson introduces the deflection coefficient C to reflect the influence of differential deflections between deckhouse and hull. Since considerable discussion was given in the text of the paper concerning the deflection coefficient, it will be sufficient to state that once this coefficient is obtained, the final expression for stress may be written as follows:

$$(21) \quad \frac{\sigma_x}{q_0} = \left[\left\{ \frac{\sigma_{x_0}}{q_0} \right\}_{y=0} + \left\{ 1 - 2C\frac{C_0}{d} \right\} \left\{ \frac{\sigma_{x_1}}{q_0} \right\}_{y=0} \right] \frac{\sigma_{x_r}}{q_0} \frac{1}{\left\{ \sigma_{x_r}/q_0 \right\}_{y=0}} + C\frac{\sigma_{x_1}}{q_0} + \left\{ 2\frac{C_0}{d} - 1 \right\} C \left\{ \frac{\sigma_{x_1}}{q_0} \right\}_{y=0}$$

where σ_{x_r} is the stress from the combined effect of the various decks, all other terms being defined previously. The above expression was derived from simple algebraic relationships relating all terms to the base of the deckhouse. It may be shown that equation (21) reduces essentially to a shearing component plus a bending component at the base, i.e.,

$$(22) \qquad \frac{\sigma_x}{q_0} = \frac{\sigma_{x_1}}{q_0} + \frac{\sigma_{x_0}}{q_0}$$

To do this let:

$$\frac{\sigma_{x_0}}{q_0} = A \quad , \qquad \left(\frac{\sigma_{x_0}}{q_0}\right)_{y=0} = A'$$

$$\frac{\sigma_{x_1}}{q_0} = B \quad , \qquad \left(\frac{\sigma_{x_1}}{q_0}\right)_{y=0} = B'$$

$$\frac{\sigma_{x_r}}{q_0} = D \quad , \qquad \left(\frac{\sigma_{x_r}}{q_0}\right)_{y=0} = D'$$

Substitution into Equation [21] yields:

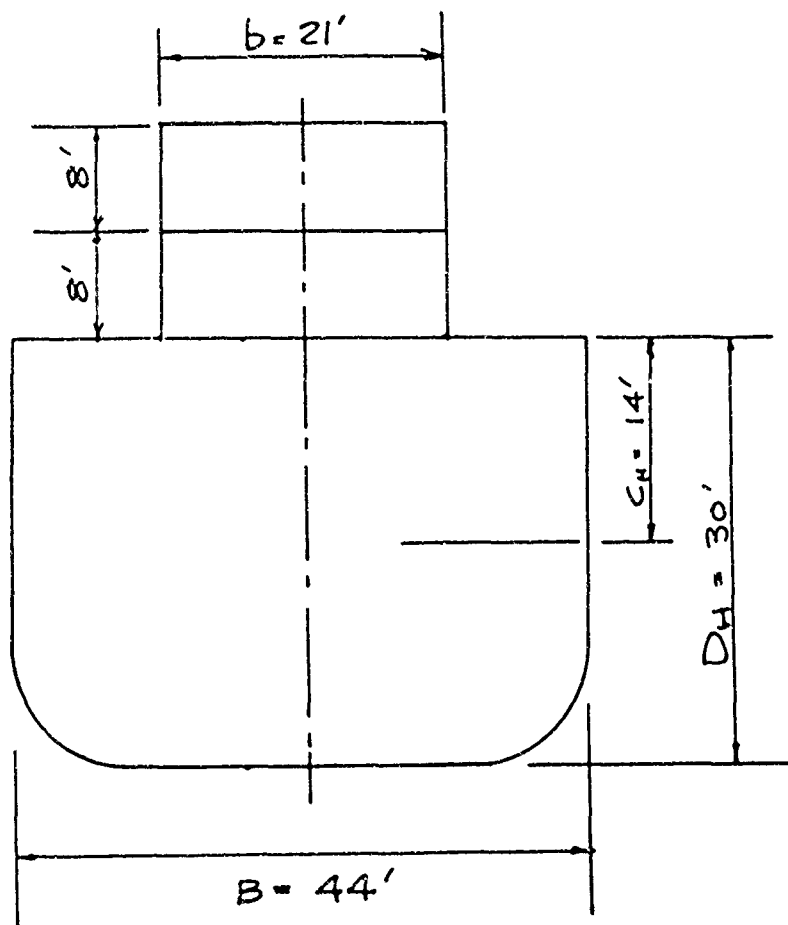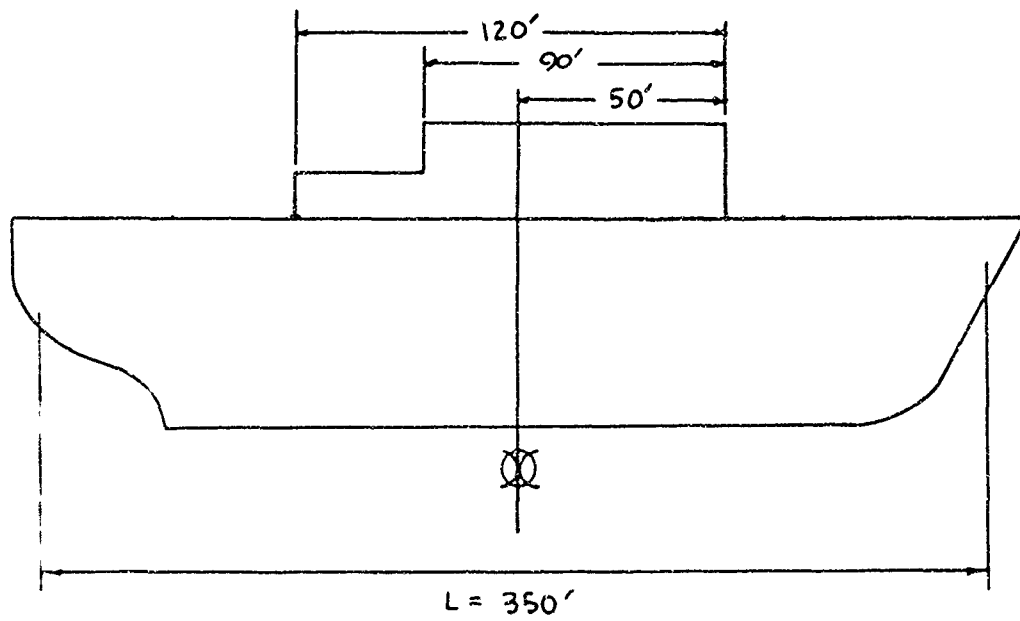$$\frac{\sigma_x}{q_0} = \left[A + (1 - 2c\frac{c_0}{d})B'\right]D\frac{1}{D'} + CB + \left(2\frac{c_0}{d} - 1\right)cB'$$

Simplifying and noticing that at the base $A, B, D = A', B', D'$ respectively we obtain

$$\frac{\sigma_x}{q_0} = A + B - 2BC\frac{c_0}{d} + CB + 2BC\frac{c_0}{d} - cB$$

$$= A + B$$

$$= \frac{\sigma_{x_0}}{q_0} + \frac{\sigma_{x_1}}{q_0} \qquad\qquad q e d$$

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APPENDIX II

DESIGN EXAMPLE

Deckhouse Material: Aluminum

Hull Material: Steel

Cross Sectional Area of Hull: $A_H = 1025$ in^2

Moment of Inertia of Hull: $I_H = 141,000$ in^2-ft^2

Maximum Bending Moment: $M = 58,000$ ft-tons

Nonsymmetry Adjustment to Deckhouse Length:

Effective length for 01-level

$50/350 = 0.143$ Therefore, from the Design Simplifications

p.19:

$$\ell_{E_{01}} = 50 + 50 = 100 \text{ FT.}$$

Effective length for 02-level

$40/350 = 0.114$ Therefore:

$$\ell_{E_{02}} = 40 + 40 = 80 \text{ FT.}$$

Since:

$$\frac{\ell_{E_{02}}}{\ell_{E_{01}}} = \frac{80}{100} = 0.80 \text{ ,}$$

the second level may be included. Therefore, the effective
length for both levels (per the Design Simplifications
p. 19) is:

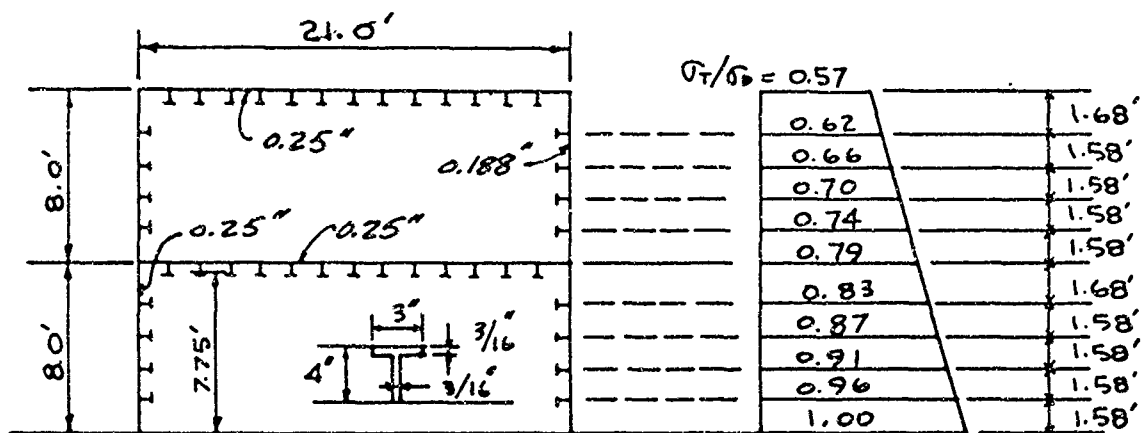$$\ell_E = \frac{80 + 100}{2} = 90 \text{ FT.}$$

Stress Ratio, σ_T/σ_D

$$\ell_E/L = 90/350 = 0.26$$

From Figure 7, for $C_H = 15$ FT. , $\sigma_T/\sigma_D = 0.57$
From Figure 8, for $C_H = 14$ FT , $K_4 = 1.00$
Therefore:

$$\sigma_T/\sigma_D = 0.57 \times 1.00 = 0.57$$

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21.0'

0.25"

0.188"

8.0'

0.25" 0.25"

8.0'

7.75'

3"

3/16

4'

3/16

$\sigma_T/\sigma_b = 0.57$

0.62	1.68'
0.66	1.58'
0.70	1.58'
0.74	1.58'
0.79	1.58'
0.83	1.68'
0.87	1.58'
0.91	1.58'
0.96	1.58'
1.00	1.58'

STRUCTURAL ARRANGEMENT
OF DECKHOUSE

STRESS - RATIO DIAGRAM

STF AREA = 1.3 IN.^{2}

SHEAR LAG FACTOR:

$b/\ell = 21/90 = 0.23$

FROM FIGURE 10, $K_3 = 0.93$

ITEM	ACTUAL AREA in²	FACTOR(S)			EQIV. AREA in²	LEVER FT.	MOMENT in²-FT.
		STRESS ×	SHEAR ×	MATERIAL			
02 - R	63.0	0.57 ×	0.93 ×	0.34	11.3	16.0	180.8
02 - DECK STF. (13)	16.9	0.57 ×	0.93 ×	0.34	3.0	15.75	47.2
02 - SIDES	36.1	0.68	—	× 0.34	8.3	11.8	97.9
02 - SIDE STF (2) (STED FROM DECK)	2.6	0.62	—	× 0.34	0.5	14.32	7.2
02 - SIDE STF (2)	2.6	0.66	—	× 0.34	0.6	12.74	7.6
02 - SIDE STF (2)	2.6	0.70	—	× 0.34	0.6	11.16	6.7
02 - SIDE STF (2)	2.6	0.74	—	× 0.34	0.7	9.58	6.7
01 - R	63.0	0.79 ×	0.93 ×	0.34	15.8	8.0	12.6
01 - DECK STF (13)	16.9	0.79 ×	0.93 ×	0.34	4.2	7.75	32.6
01 - SIDES	48.0	0.89	—	× 0.34	14.4	3.8	54.7
01 - SIDE STF (2) (LISTED FROM DECK)	2.6	0.83	—	× 0.34	0.7	6.32	4.4
01 - SIDE STF (2)	2.6	0.87	—	× 0.34	0.8	4.74	3.8
01 - SIDE STF (2)	2.6	0.91	—	× 0.34	0.8	3.16	2.5
01 - SIDE STF (2)	2.6	0.96	—	× 0.34	0.9	1.58	1.4
				$A_{DE} =$	62.6	13.2	826.3
						$m_{DE} =$	1292.4

$$c_E = \frac{A_H \, c_H}{A_H + A_{DE}} = \frac{(1025)(14)}{1025 + 62.6} = 13.2 \text{ ft.}$$

$$I_E = I_H + A_H (c_H - c_E)^2 + m_{DE} \, c_E$$

$$= 141,000 + 1025(14.0 - 13.2)^2 + 1292.4(13.2)$$

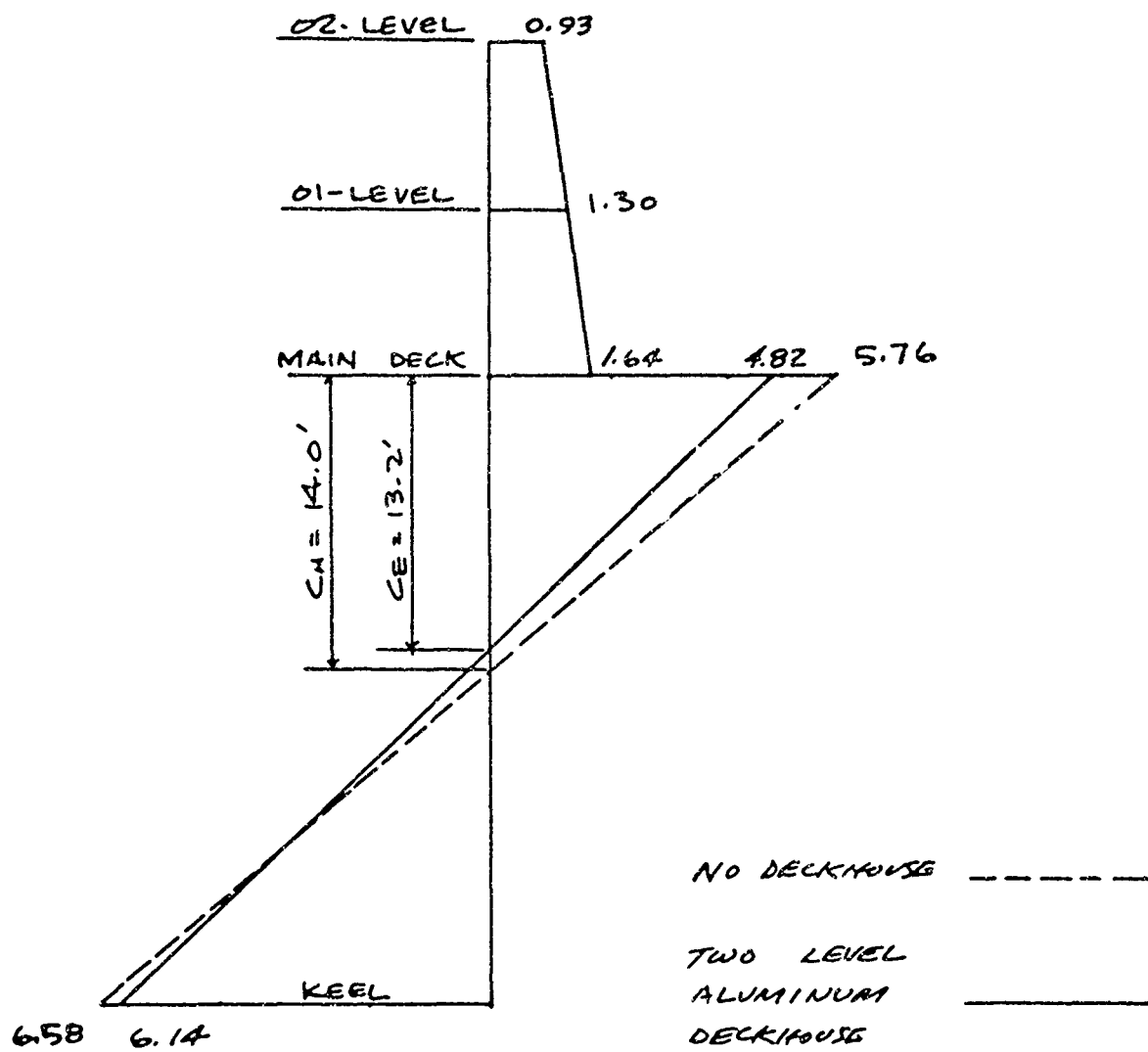$$= 158,716 \quad \text{in}^2 - \text{ft}^2$$

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STRESSES:

AT STRENGTH DECK :
MATERIAL : STEEL
$$\sigma_D = \frac{M\,c_E}{I_E} = \frac{58,000 \times 13.2}{158,716} = 4.82 \text{ TSI}$$

AT BASE OF DECKHOUSE :
MATERIAL : ALUMINUM
$$\sigma_D = 4.82 \times 0.34 = 1.64 \text{ TSI}$$

AT DECKHOUSE TOP :
MATERIAL : ALUMINUM
$$\sigma_T = \left(\frac{\sigma_T}{\sigma_D}\right) \cdot \sigma_D = 0.57 \times 1.64 = 0.93 \text{ TSI}$$

AT 01- LEVEL :
MATERIAL , ALUMINUM
$$\sigma_{01} = 0.79 \times 1.64 = 1.30 \text{ TSI}$$

AT KEEL :
MATERIAL : STEEL
$$\sigma_K = \frac{M\,(D_H - c_E)}{I_E} = \frac{58,000\,(30.0 - 13.2)}{158,716}$$

$$= 6.14 \text{ TSI}$$

02. LEVEL 0.93

01-LEVEL 1.30

MAIN DECK 1.64 4.82 5.76

$C_N = 14.0'$

$C_E = 13.2'$

KEEL

6.58 6.14

NO DECKHOUSE — — — — —

TWO LEVEL
ALUMINUM ————
DECKHOUSE

STRESS DISTRIBUTION

41
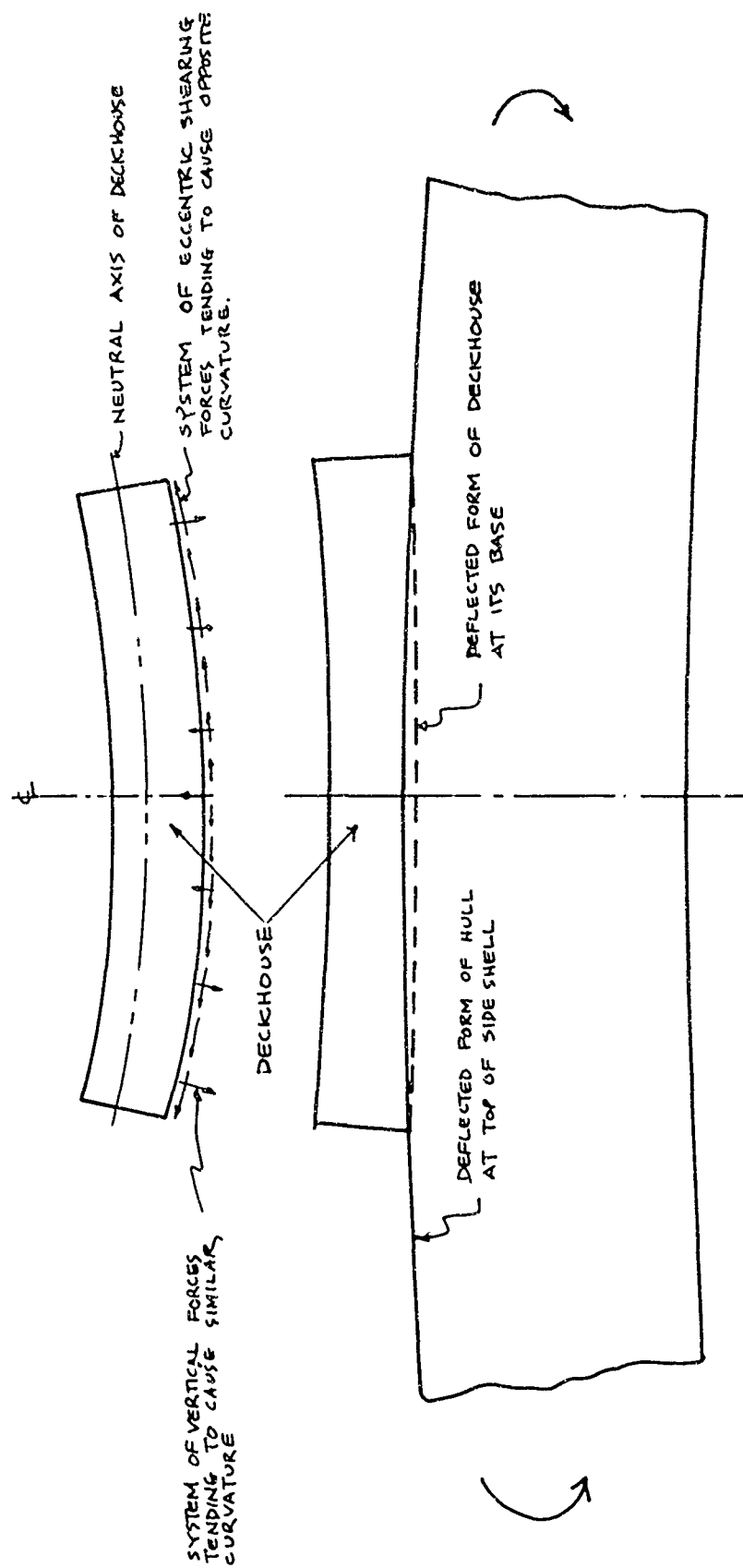
FIGURE 1

The systems of vertical and shearing forces which affect the curvature of the deckhouse.

THE DISTRIBUTION OF LONGITUDINAL STRESSES IS WANTED HERE

$\ell/2$

x

$L/2$

A SYSTEM OF STRESSES IS APPLIED AT THE BASE SUCH THAT VERTICAL & LONGITUDINAL DISPLACEMENTS ARE COMPATIBLE WITH THOSE PRODUCED BY FLEXURE OF THE HULL
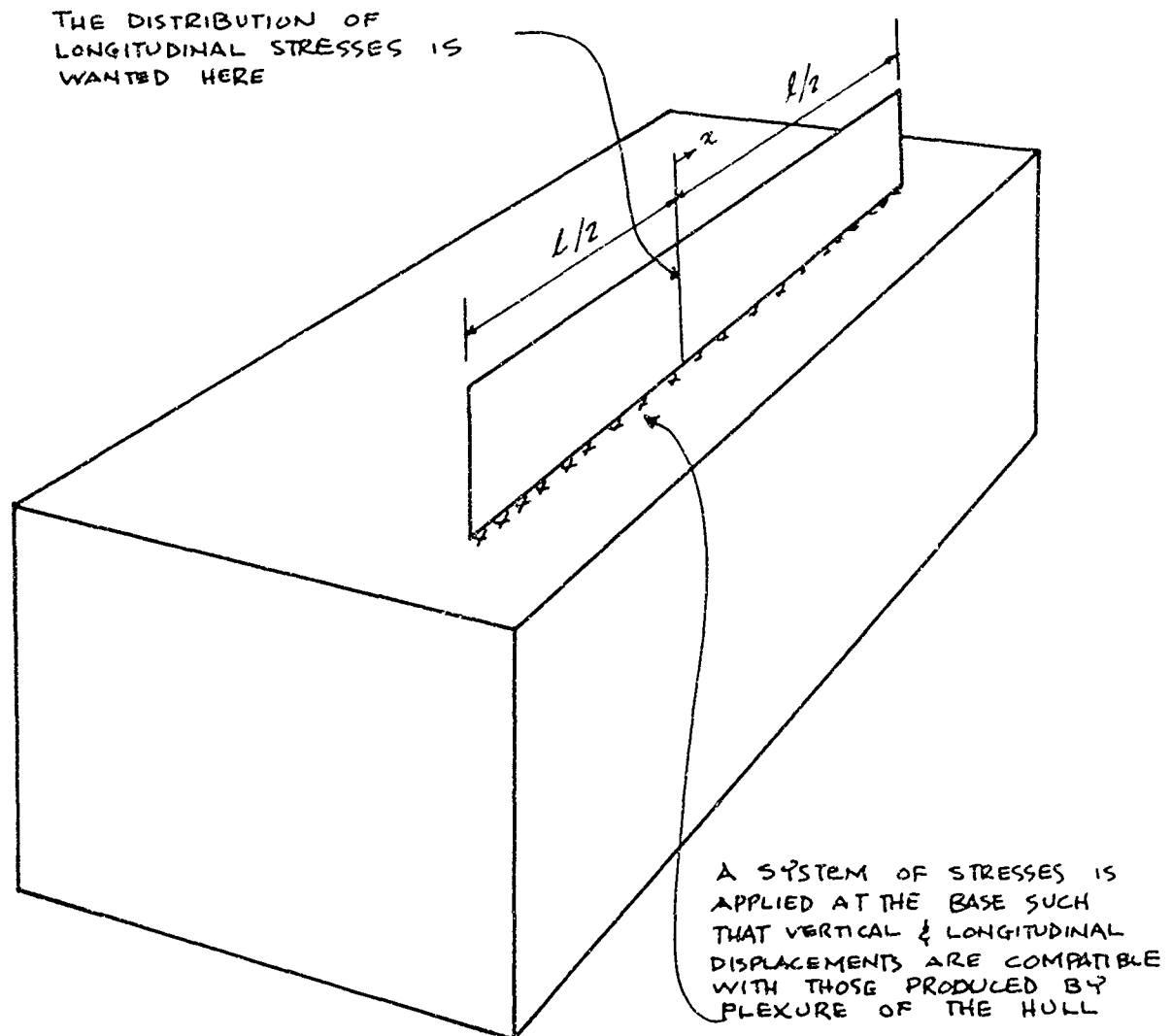
FIGURE 2

"The approach is to use the Airy Stress Function to represent the stress in a rectangular plate, which is attached to the hull and is analogous to the deckhouse side."
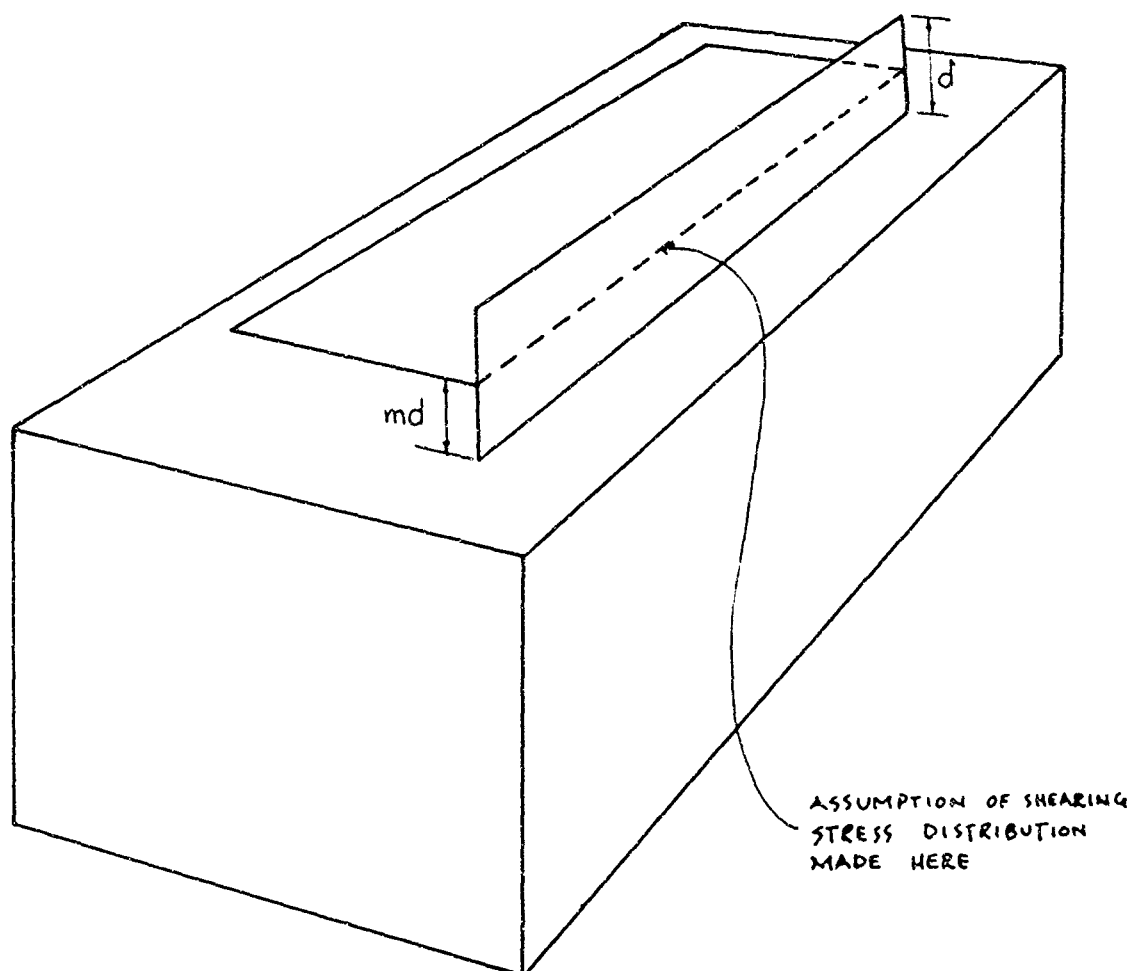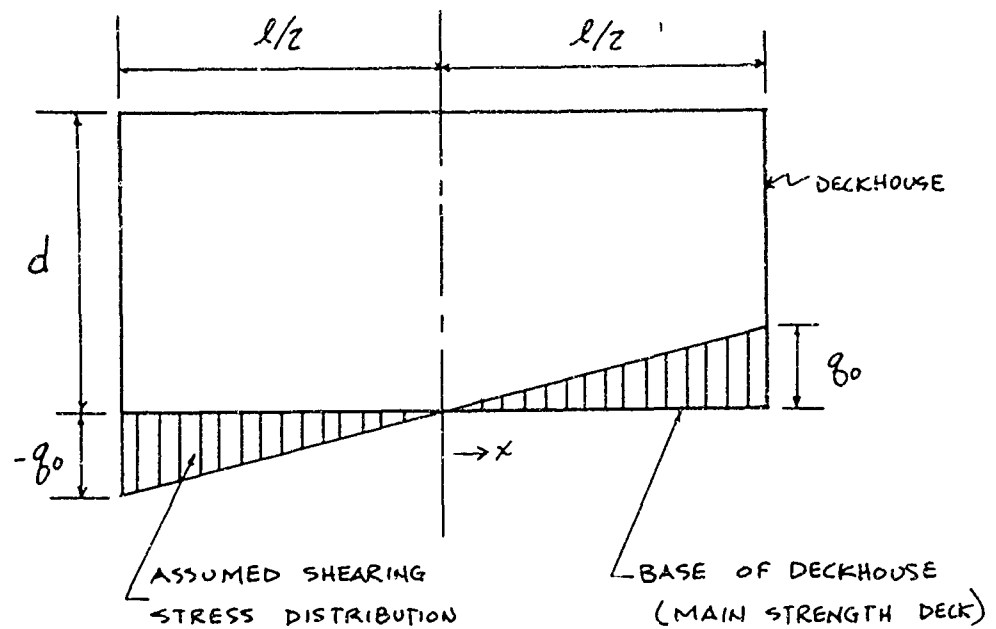
Labels on figure: d, md, ASSUMPTION OF SHEARING STRESS DISTRIBUTION MADE HERE

FIGURE 3

"Consideration is then given to the effect on the
stress caused by attaching a plate to the deck-
house side. The attached plate represents a deck."
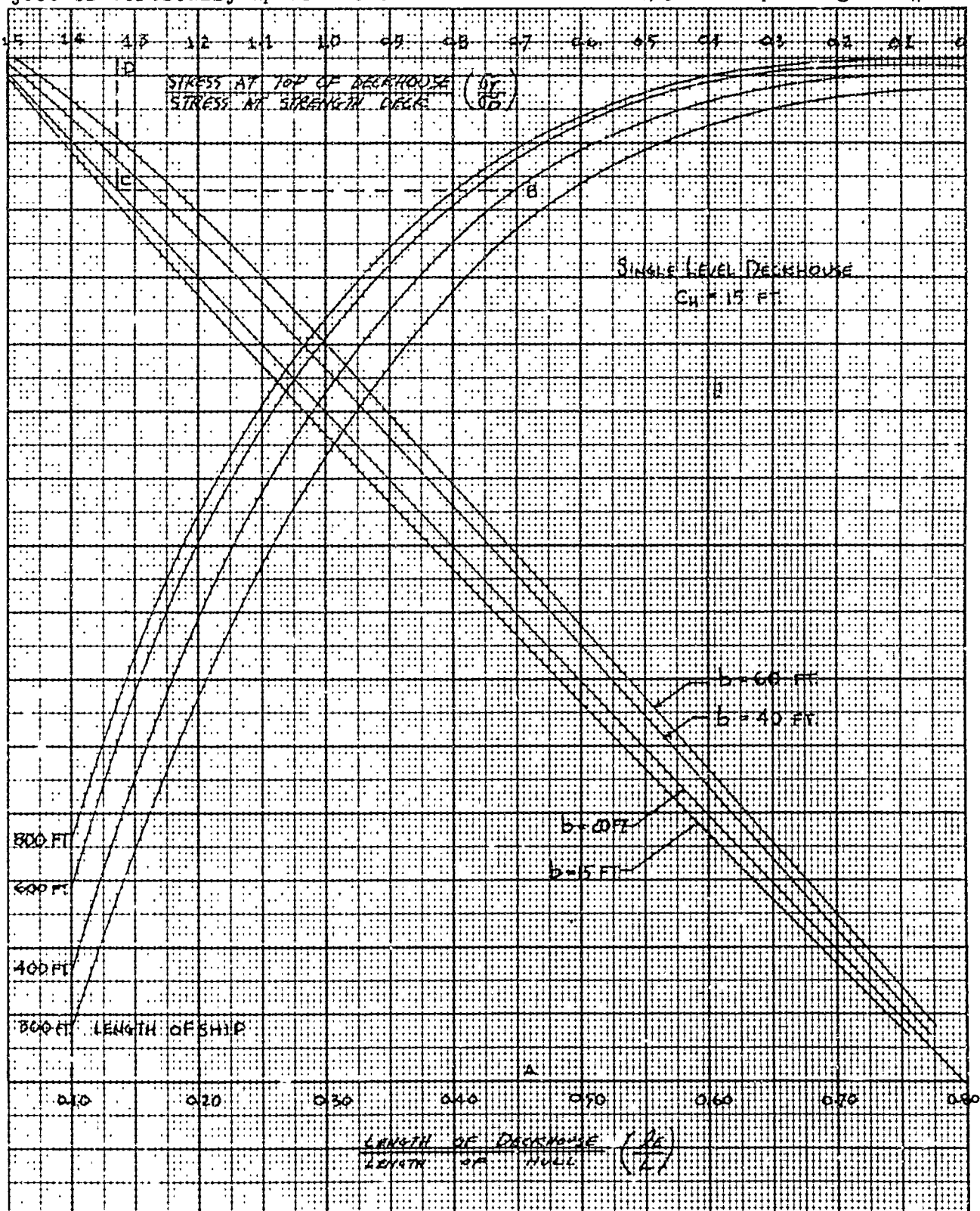
$$\tau_{xy} = q_0 \sum_1^n \beta \sin \alpha_n x$$

$$\beta = \frac{8}{\pi^2}\left(\pm \frac{1}{n^2}\right) \qquad (n = 1, 3, 5 \ldots)$$

FIGURE 4

"The basic assumption used throughout Johnson's
analysis is that the shearing stress distribution
in the deckhouse at the connection to any deck
varies linearly along the length of the deckhouse."

FIGURE 5

Stress ratio for single level deckhouse, with neutral axis of hull 15 feet
below strength deck. Erect ordinate AB from the given value of ℓ_f/L to the
curve corresponding to ship length, L (interpolating as necessary). Draw a
horizontal line BC to the curve for the given breadth of deckhouse, b. Pro-
ject CD vertically upward to obtain the value of $6_T/6_D$ corresponding to $C_H = 15$ FT

FIGURE 6

Correction to Stress Ratio for Single Level
Deckhouse for location of neutral axis of
hull. Enter diagram at given value of l/L,
interpolating to suit the given length
ratio, l/L. Multiply the correction factor
K_2 by the value of σ_1/σ_b obtained from
Figure 5.

K_2 (CORRECTION FACTOR FOR FIGURE 5)

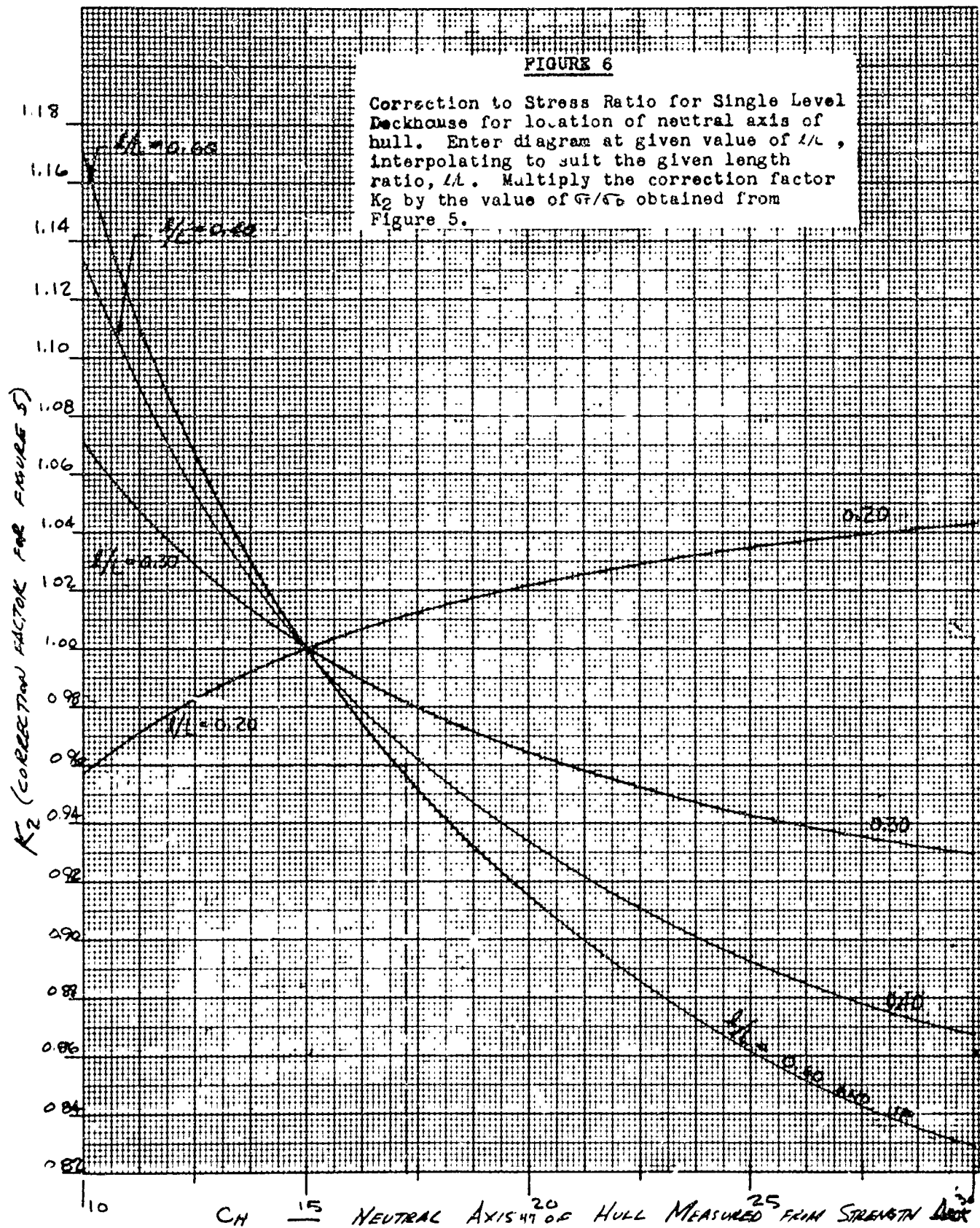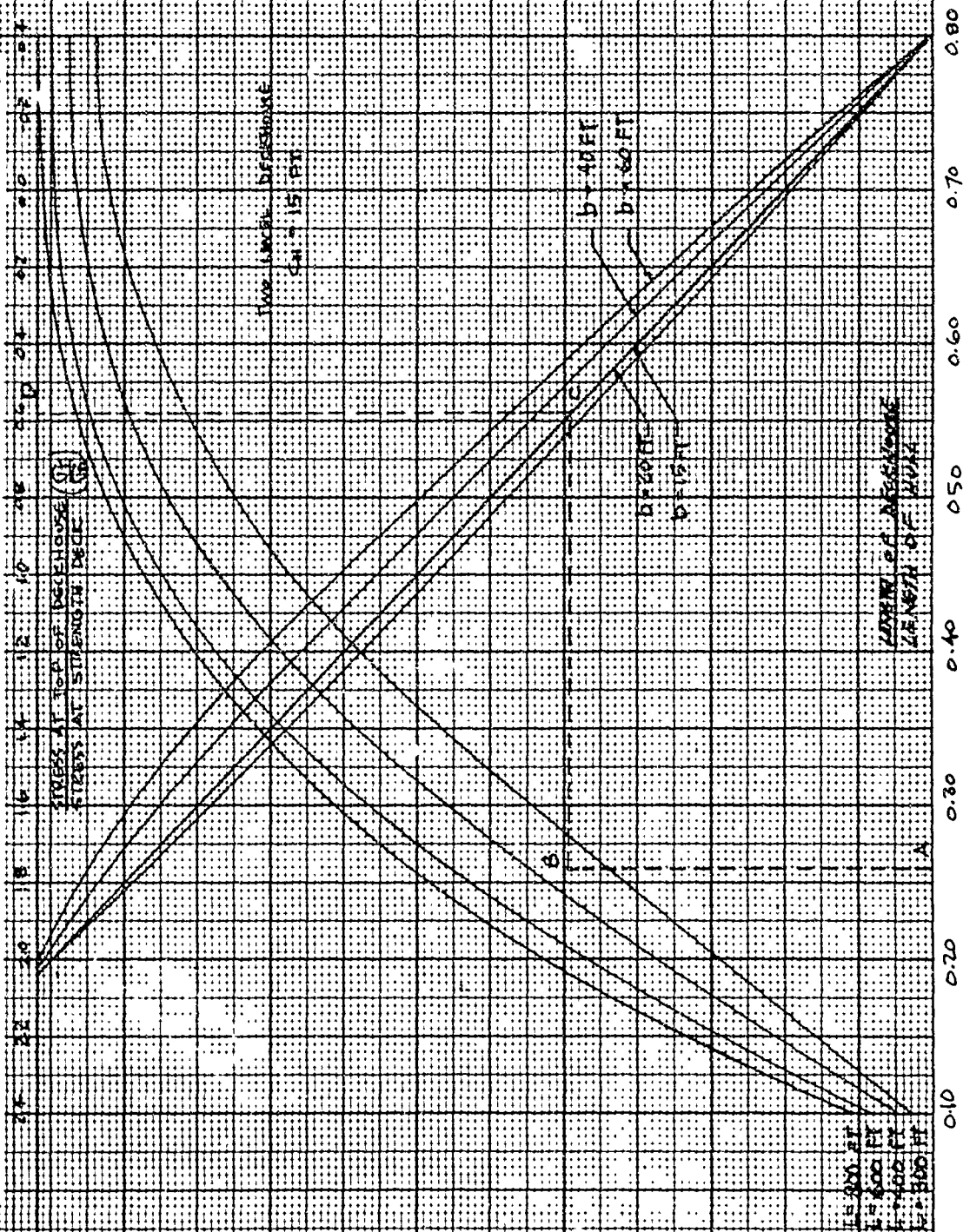CH — NEUTRAL AXIS 47 OF HULL MEASURED FROM STRENGTH DECK

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FIGURE 7

Stress Ratio for Two Level Deckhouse. Erect ordinate AB from the given value of ℓ/L to the curve corresponding to ship length, L (interpolating as necessary). Draw a horizontal line BC to the curve for the given breadth of deckhouse, b. Project CD vertically to obtain the value of $\frac{\sigma_L}{\sigma_D}$
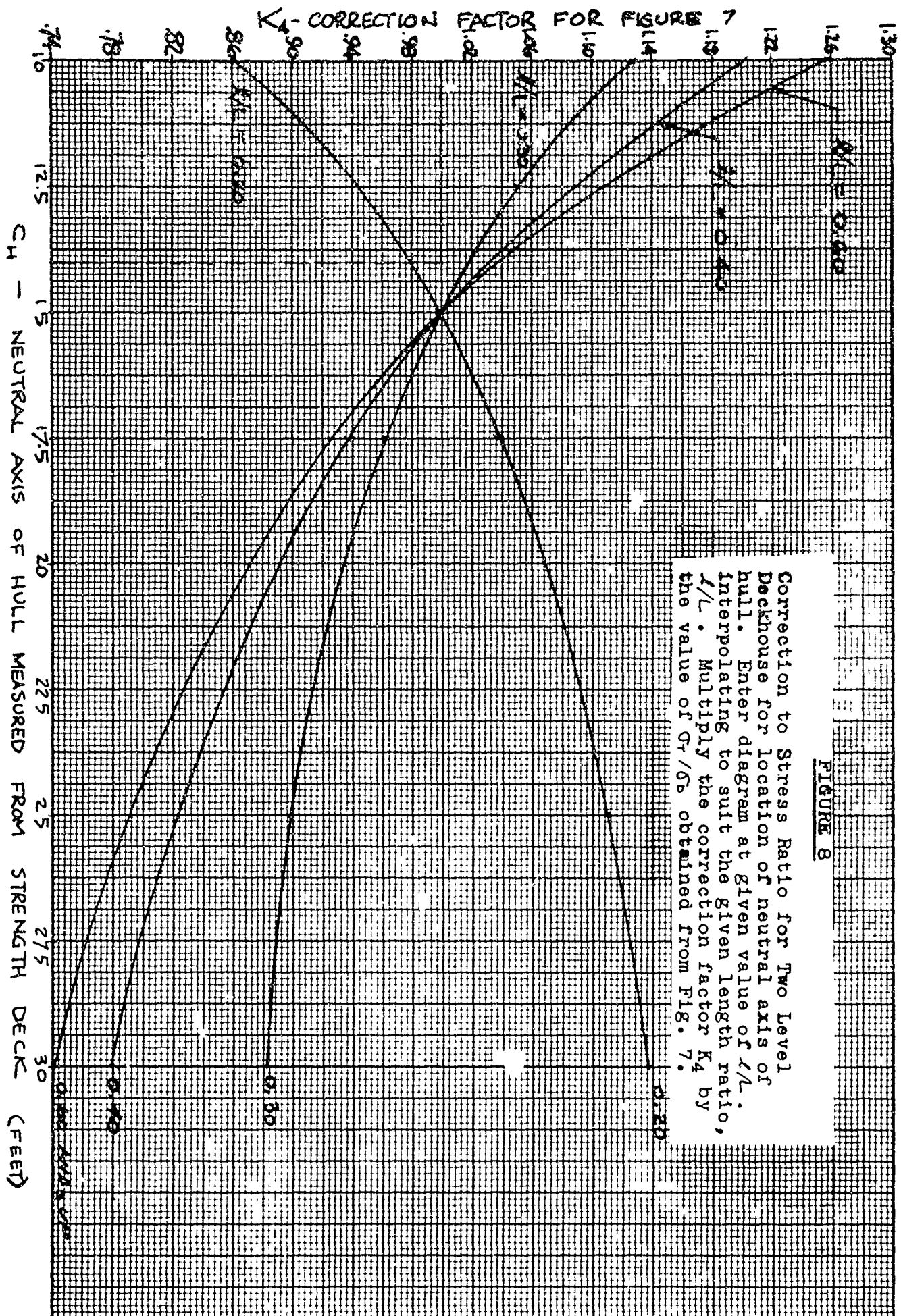
FIGURE 8

Correction to Stress Ratio for Two Level Deckhouse for location of neutral axis of hull. Enter diagram at given value of l/L, interpolating to suit the given length ratio, l/L. Multiply the correction factor K_4 by the value of σ_r/σ_b obtained from Fig. 7.

Figure legend:

σ_T - STRESS AT TOP OF DECKHOUSE
σ_D - STRESS AT STRENGTH DECK
σ_K - STRESS AT KEEL
σ - STRESS AT ANY POINT IN DKHOUSE
σ' - STRESS AT ANY POINT IN HULL
C_E ≈ EFF. NEUTRAL AXIS (HOUSE & HULL)
C_D - NEUTRAL AXIS OF DECKHOUSE
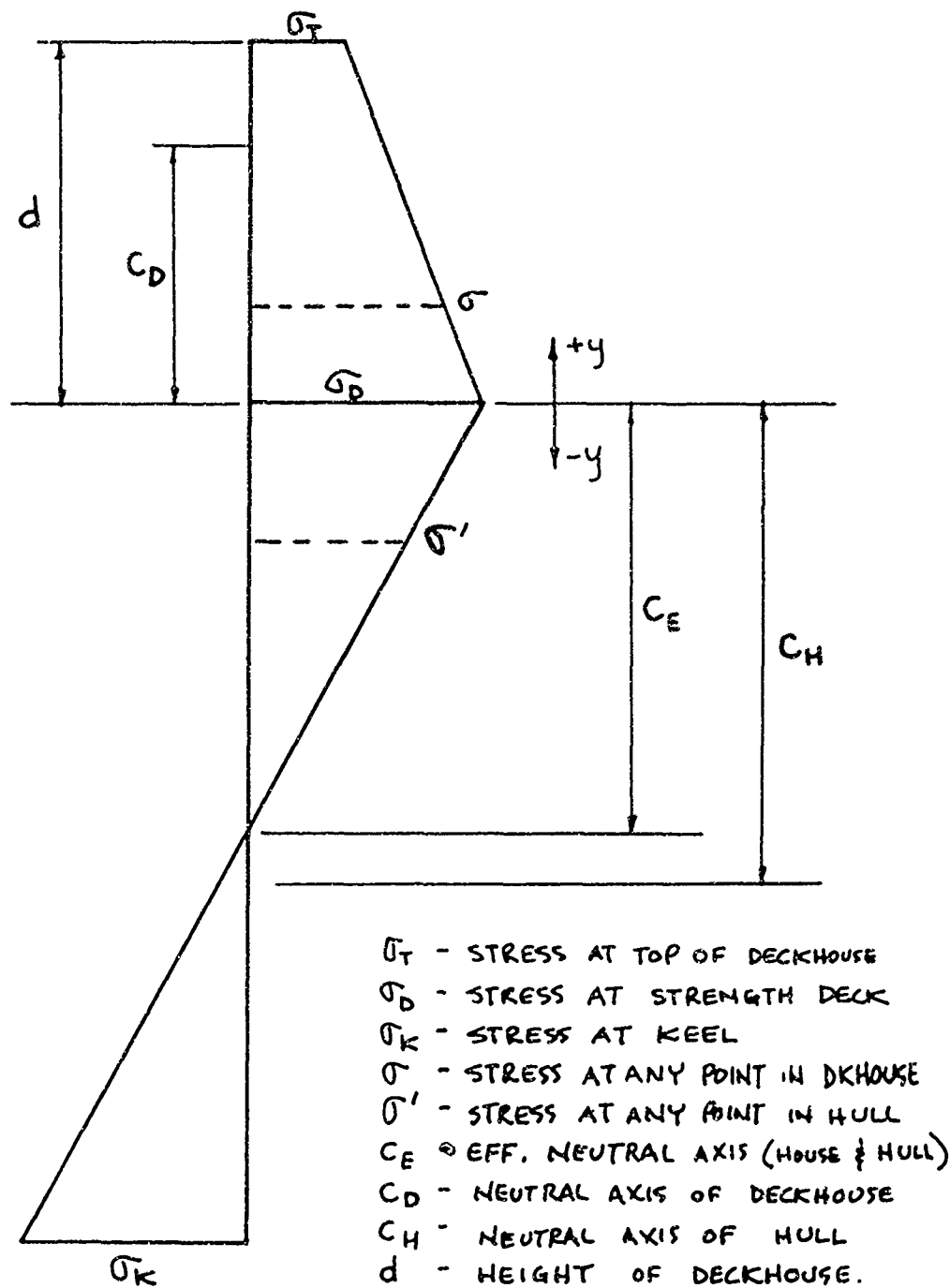C_H - NEUTRAL AXIS OF HULL
d - HEIGHT OF DECKHOUSE.

FIGURE 9

"The approximation of linear stress in the deckhouse based on
Johnson's solution for σ_T/σ_D. It was found expeditious at this
point to rationalize a linear distribution of stress between
the possibly nonlinear values of stress at the strength deck
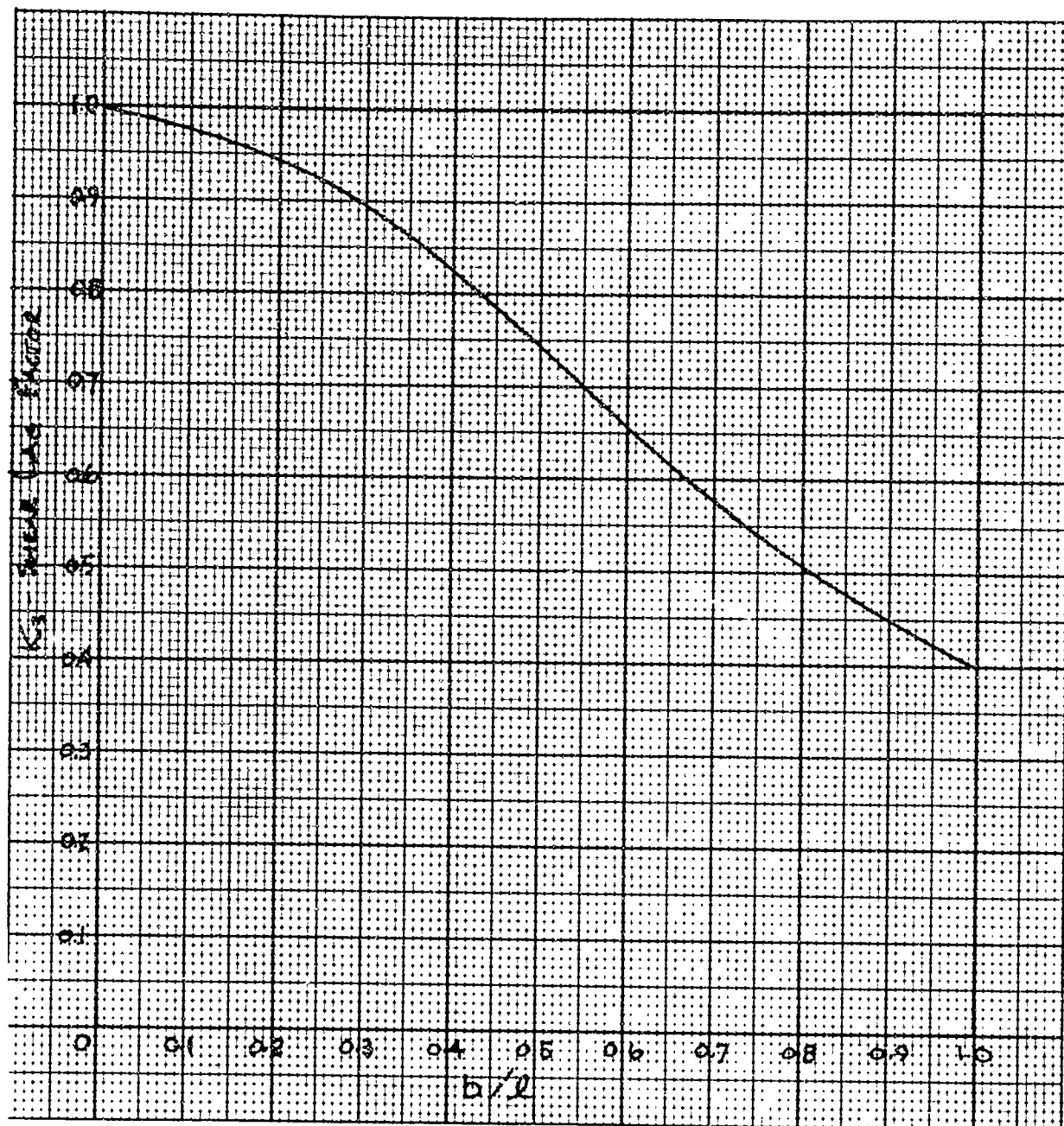and top os deckhouse."

FIGURE 10

Shear Lag Factor. Multiply the actual areas
of plating and stiffeners by the factor cor-
responding to the given ratio of b/ℓ. This
allows for the variation in stress across
the breadth of the deckhouse due to "shear
lag".

SUMMARY

ℓ/L	C_E		I_E		σ_T/σ_D		σ_D		$\sigma_{01\text{-}LEV}$		σ_T	
	1-LEVEL	2-LEVELS	1-LEVEL	2-LEVELS	1-LEVEL	2-LEVELS	1-LEVEL	2-LEVELS	2-LEVELS	1-LEVEL	1-LEVEL	2-LEVELS
0.10	14.0	13.9	170,800	173,410	0.4	0	4.9	4.8	2.2	2.4	2.0	0
0.35	13.3	12.0	185,590	224,300	1.2	1.2	4.3	3.2	3.3	2.5	5.2	3.85
0.80	12.9	11.1	194,416	252,245	1.5	2.0	4.0	2.6	3.9	3.0	6.0	5.2
BEAM THEORY RESULTS	12.9	10.5	195,035	268,046	1.6	2.6	4.0	2.5	4.1	3.1	6.4	5.9

FOR NO DECKHOUSE MATERIAL
INCLUDED IN STRENGTH CALCULATION:

$C_E = C_H = 15$ FT.
$I_E = I_H = 150,000$ $IN^2 \cdot FT^2$
$\sigma_D = 6.0$ T.S.I.

ASSUMED:

$A_H = 1000$ IN^2
$M = 60,000$ FT-TONS
$L = 400$ FT.
$b = 20$ FT.
$d = 8$ FT., 16 FT.

FIGURE 11

"From the summary, it may be seen that the consideration of two levels at any particular value of ℓ/L will provide lower allowable design stresses in both hull and deckhouse. Therefore it is conservative to disregard an upper level." (Last row reflects the results of beam theory applied to deckhouse and hull as a single unit.)

$$\frac{\partial^2 v}{\partial x^2} \cong \frac{1}{R}$$

$$\epsilon_x = \frac{\partial u}{\partial x}$$

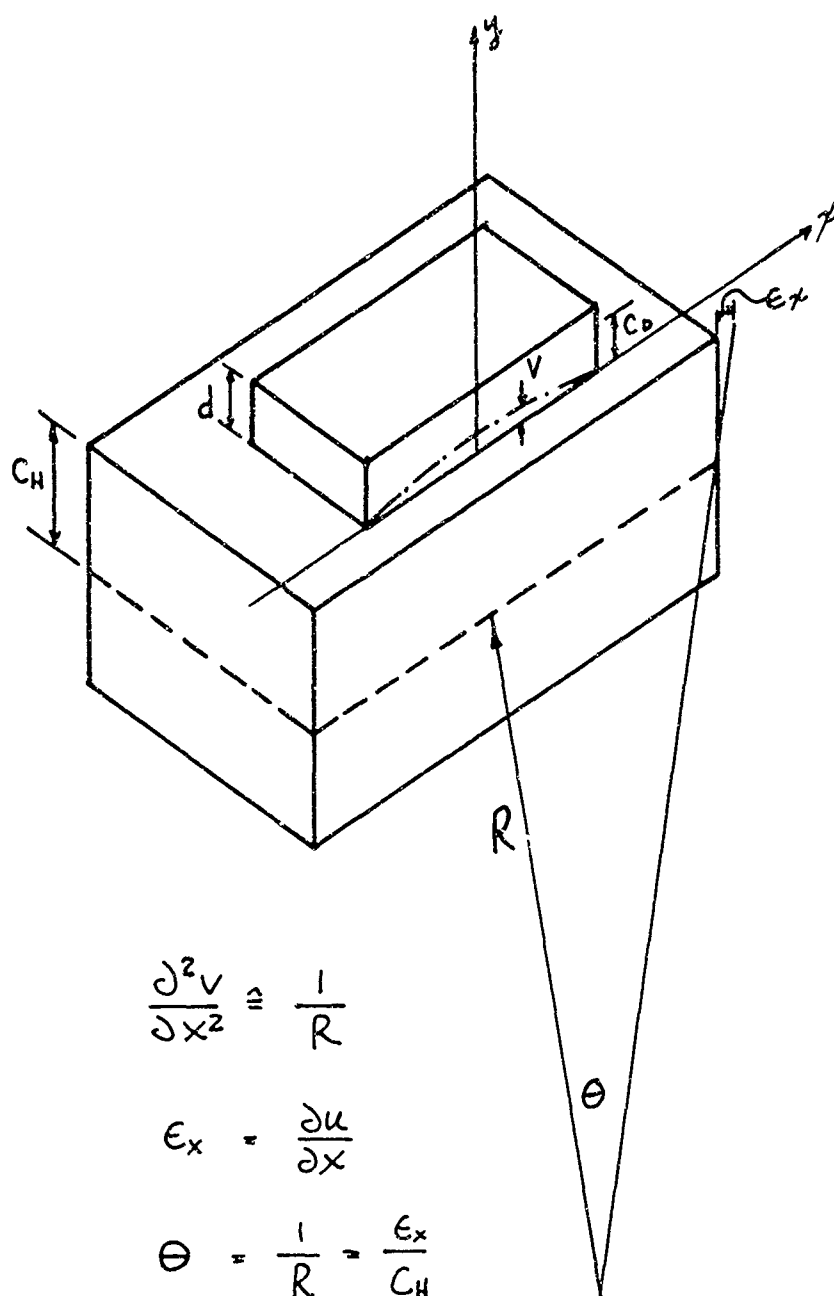$$\theta = \frac{1}{R} = \frac{\epsilon_x}{C_H}$$

FIGURE 12

"Simple beam relations. In this part of the analysis,
the deflections at the base of the deckhouse are
assumed equal to those of the main hull girder.
Later the effect of differential deflections between
deckhouse and hull will be brought into the analysis."